

Equilibrium Fast Trading

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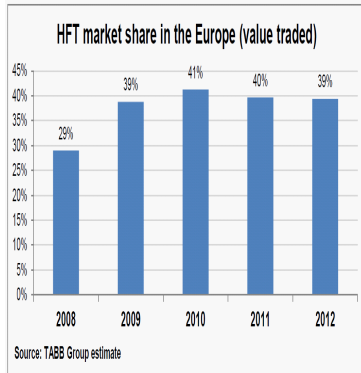
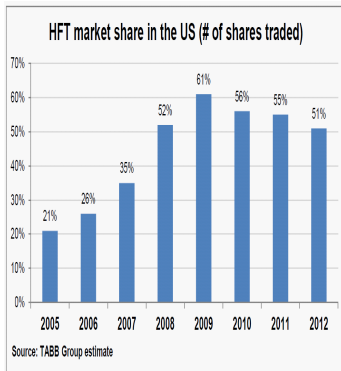
²HEC Paris

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Financial Innovations

- **Financial Innovations : New ways to share risks :**
 1. New assets/instruments (e.g., options, futures, swaps, CDS etc.)
 2. New markets : exchanges, trading technologies...
- **High frequency trading is one key trading innovation of the last few years.**

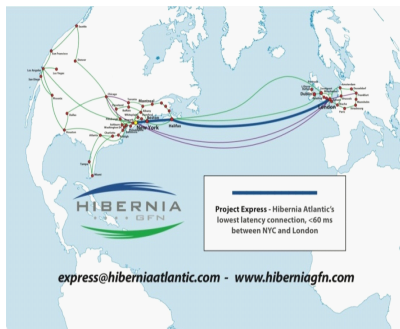
High Frequency Trading



- **No agreed definition on what HFT is.** SEC (2010) : *The use of extraordinarily high-speed and sophisticated computer programs for generating, routing, and executing orders [...]*

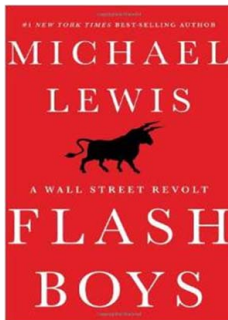
Investment in fast trading technologies

- **Example** : Project Express : Transatlantic cables to reduce transmission time between the City and Wall Street from 64.98 milliseconds to 59.6 milliseconds. Cost : \$300 mio.



- **Others** : Microwave radio transmission, etc. Tabb Group estimate of investment in fast trading technologies in 2012 : \$1.5 bn.
- **Socially Optimal? Good or bad financial innovation?**

Highly controversial



THE NEW YORK TIMES

Thursday, September 25, 2014

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1 FLASH BOYS, by Michael Lewis.
(Shorts.) The world of high-frequency computer-driven trading.

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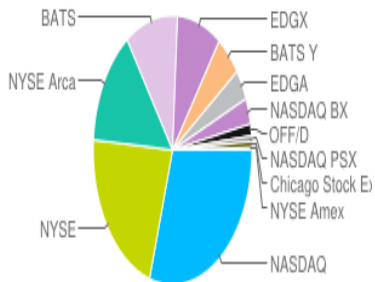
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Market fragmentation




- More than 50 different trading venues for U.S. stocks (including 13 registered exchanges)
- Same situation in Europe (with fewer markets)
- Spotting attractive quotes is difficult in this environment
⇔ search problem.

Information



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BATS SPY 

Market Quality Statistics¹

SPDR S&P 500 ETF TR TR UNIT

Orders Accepted: 1,174,665 Total Volume: 10,639,379

TOP OF BOOK			LAST 10 TRADES		
	SHARES	PRICE	TIME	PRICE	SHARES
ASKS ↑	32,200	180.91	12:48:39	180.86	100
	34,100	180.90	12:48:38	180.86	300
	44,650	180.89	12:48:38	180.86	100
	31,600	180.88	12:48:38	180.86	300
	34,900	180.87	12:48:38	180.86	200
BIDS ↓	100	180.86	12:48:38	180.86	100
	4,413	180.85	12:48:38	180.86	100
	10,000	180.84	12:48:38	180.85	100
	31,800	180.83	12:48:37	180.85	47
	31,300	180.82	12:48:37	180.85	53

Last updated 12:48:43

Adverse Selection

1. Mounting evidence that high frequency traders' market orders are informed (e.g., Brogaard, Hendershott, and Riordan, RFS, 2014).

High-frequency traders [...] see where the market is moving before other investors can see or act on it [...] We have exchanges [...] selling to high-frequency traders a faster feed. If they are getting the fast feed, they are able to see the price of IBM is moving higher before the ordinary investor does [...] and [...] can only win by making the rest of those in the market losers.

[“Is Wall Street Pulling a Fast One”, Newsweek, June 2014.]

Why do traders use fast trading technologies ?

- **Navigate more efficiently in fragmented markets** : realize gains from trade (share risks) more quickly.
 1. Opportunity costs of delayed and missed trades are high in financial markets (see Chiyachantana and Jain (2009))
- **React faster to new information** : market data (quote updates, trades, orders) and machine readable information (newswires, tweet, blogs, etc.).

Our Objective 1/2

- **Fast trading technologies = information technologies** but provide simultaneously two types of information :
 1. Information on quotes
 2. Information on future price movements.
- We develop a model that captures this dual role of fast trading technologies. We use the model to analyze :
 1. **Equilibrium level of investment in fast trading technologies.**
 2. **Welfare** : Is investment in fast trading technologies socially efficient or not
 3. **Regulatory issues** : Should one separate slow and fast ? Should one tax high frequency traders ? How ?

Our Objective 2/2

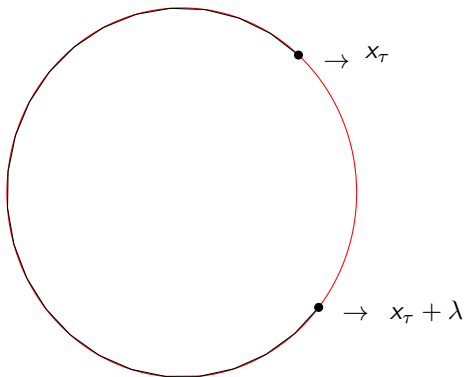
The topic of financial innovation is a very broad one. It requires nothing less than a positive theory of endogenous market structure (i.e., which markets, institutions, and instruments will be introduced and which will not) together with an analysis of the welfare economics of existing market structures (Allen and Gale (1994), “Financial Innovation and Risk Sharing”)

Model

- **One risky asset that trades at dates** : $\tau \in \{1, \dots, t, \dots, \infty\}$
- The cash flow of asset τ at the end of period τ is $\theta_\tau \in \{-\sigma, +\sigma\}$ with equal probabilities.
- Cash-Flows are i.i.d. with mean zero.
- Risk free asset : returns r .
- **The market for this asset is fragmented : it trades in a continuum of trading venues.**

Market Fragmentation

- All markets in $[x_\tau, x_\tau + \lambda]$ are liquid (post quotes). x_τ is random (new draw at each date) and uniformly distributed on $[0, 1]$.



Financial Institutions

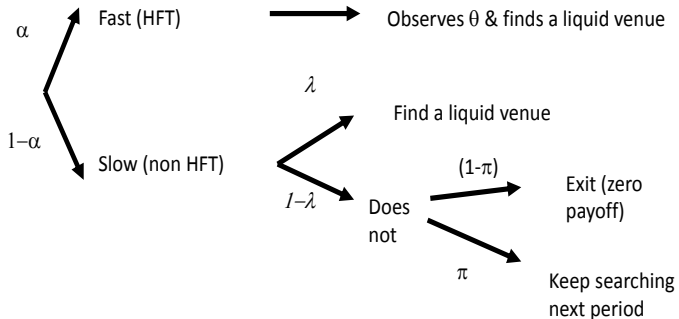
- **In each period, a continuum of risk neutral financial institutions enter the market.**
 1. They can buy or sell one share of the asset if they find a liquid market or do nothing.

- **An institution with private valuation δ values the date t cash-flow at :**

$$\underbrace{\theta_t}_{\text{Common Value}} + \underbrace{r(1+r)^{-1}\delta}_{\text{Private Value}}$$

- **Heterogeneity in private values** (e.g., due to differences in tax treatments, regulatory requirements, or hedging needs; standard in the literature : see Duffie, Garleanu and Pedersen (2005, Eca) \implies Gains from trade
- Private valuations are i.i.d across institutions with cumulative probability distribution $G(\cdot)$.

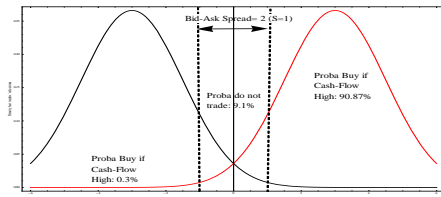
Fast and Slow Institutions



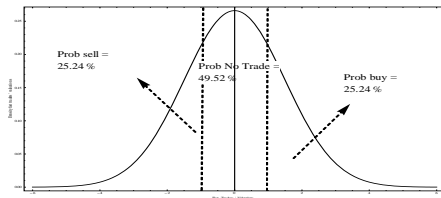
Trading

- **Quotes in each liquid trading venue are posted by risk neutral dealers ($\delta = 0$).**
 1. All institutions' buy orders execute at Ask = S
 2. All institutions' sell orders execute at Bid = $-S$
 3. The half bid-ask spread, S , is set competitively by dealers (make zero expected profit on average).
- **An institution optimally buys if its valuation for the asset exceeds S , sells if it is less than $-S$, and does not trade otherwise.**
 1. Fast institutions entering at date τ values the asset at $\theta_\tau + \delta$.
 2. Slow institutions values the asset at δ .

Example : Normal Distribution for Institutions' Private Valuations



In Red (black) : Distribution of fast institutions's valuations conditional on the asset cash flow-being high (low)



Distribution of slow institutions' valuations

Remarks

- **Similar to Grossman/Stiglitz (1980) but :**
 1. All traders make optimal decisions (no noise traders) : welfare and policy analysis is possible.
 2. Information is about asset payoffs **and** trading opportunities.

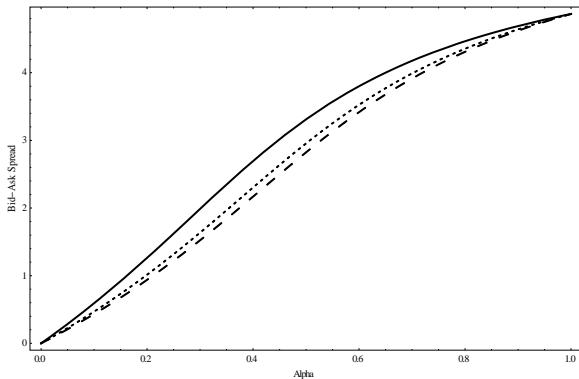
Adverse Selection, Fast Trading, and Trading Volume

- Dealers' expected profit per capita :

$$Volume \times S - \underbrace{\left| \text{Dealer's Net Position} \right| \times \text{Cash} - \text{Flow Volatility}}_{\text{Adverse Selection Cost}}$$

- Equilibrium spread** : S^* such that dealers' expected profit is zero.
- If $\alpha > 0$: Dealer are more likely to receive orders in the direction of the asset cash-flow \implies build up inventory in the "wrong" direction \implies Adverse Selection cost > 0
- Consequence** : Equilibrium spread increases with the level of fast trading (α).
- Prediction** : The informational content of trades ("permanent price impact") should increase with the level of fast trading (α).
- Consistent with Brogaard, Hendershott and Riordan (2014, RFS)'s findings.

Example (Private valuations are normally distributed)



Equilibrium Profits of Fast and Slow Traders 2/2

- Let $\omega(\delta, \theta_\tau) \in \{-1, 0, 1\}$ be the trading decision of a fast institution with private valuation δ and information θ_τ . Similarly, $\omega(\delta, 0) \in \{-1, 0, 1\}$ is the trading decision of a slow institution.
- Fast Institutions' ex-ante expected profit :**

$$\phi(\alpha) = 2E([\delta + \theta_\tau - S^*(\alpha)]\omega(\delta, \theta_\tau))$$

- Slow Institutions' ex-ante expected profit :**

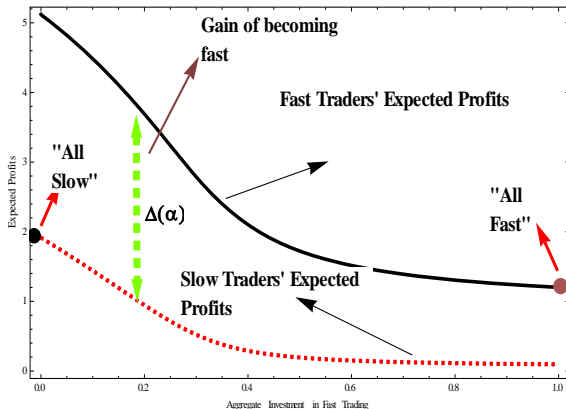
$$\psi(\alpha) = \mu(\lambda, \pi, r)E([v(\delta, 0) - S^*(\alpha)]\omega(\delta, 0)),$$

where

$$\mu(\lambda, \pi, r) = \sum_{t=\tau}^{t=\infty} \left(\frac{(1-\lambda)\pi}{1+r} \right)^{t-\tau} \lambda = \frac{\lambda}{(1 - (1-\lambda)\pi(1+r)^{-1})}.$$

- $\mu(\lambda, \pi, r) < 1$ if $\pi < 1$ or/and $r > 0$ = Waiting is Costly=Search cost.

Example : Fast and Slow Traders' Expected Profits



Fast Trading Externality

- **Result : Fast institutions obtain a higher expected profit than slow institutions but an increase in the level of fast trading makes all institutions worse off.**
- **⇒ Negative Externality of Fast Trading :** The decision to be fast makes an institution better off but makes all other institutions worse off.
- **Why ?**
 1. An increase in α raises **adverse selection**
 2. **⇒** Increase in the bid-ask spread
 3. **⇒** lower trading revenues for all institutions + increased probability of no trading (welfare loss).

Equilibrium Fast Trading

- Now consider the decision to invest or not in the fast trading technology . Let C be the cost of the technology. Investing is optimal if :

$$\underbrace{\phi(\alpha) - C}_{\text{Net Profit Fast}} \geq \underbrace{\psi(\alpha)}_{\text{Profit Slow}} ,$$

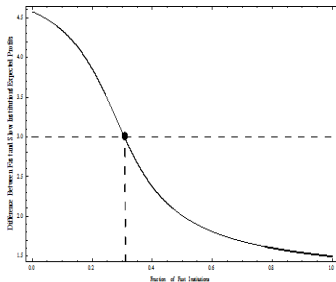
- That is, if :

$$\underbrace{\phi(\alpha) - \psi(\alpha)}_{\text{Relative Value of Fast Trading}} \geq \underbrace{C}_{\text{Cost of Fast Trading}} ,$$

- Equilibrium level of fast trading** : α^* such that :

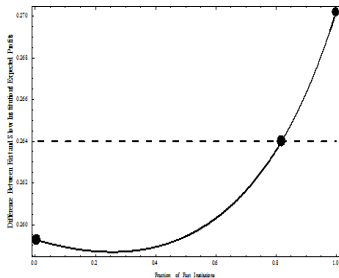
- $\alpha^* = 1$ if $\phi(1) - \psi(1) > C$.
- $\alpha^* \in (0, 1)$ if $\phi(\alpha^*) - \psi(\alpha^*) = C$.
- $\alpha^* = 0$ if $\phi(0) - \psi(0) < C$.

Arms' Race



Case 1 : Investment decisions are "substitutes" : more fast trading makes fast trading less attractive → Equilibrium fast trading level is unique.

- Which case obtains depends on the distribution of institutions' valuations.



Case 2 : "Arms Race :" Investment decisions are "complements" : more fast trading makes fast trading more attractive → Multiple equilibria.

Welfare

- Is the equilibrium level of investment in fast trading the right one for society ?
- Market participants' average welfare ("Utilitarian Welfare") :

$$W(\alpha) = \underbrace{\alpha(\phi(\alpha) - C)}_{\text{Fast Welfare}} + \underbrace{(1 - \alpha)\psi(\alpha)}_{\text{Slow Welfare}}.$$

- Let α^{SOC} be the socially optimal level of investment in fast trading. It maximizes $W(\alpha)$.

Social vs. Private Costs

- If $\alpha^{SOC} \in (0, 1)$, it solves :

$$\frac{\partial W(\alpha)}{\partial \alpha} = \phi(\alpha^{SOC}) - \psi(\alpha^{SOC}) - C$$

$$+ \underbrace{\left[\alpha^{SOC} \frac{\partial \phi(\alpha^{SOC})}{\partial \alpha} + (1 - \alpha^{SOC}) \frac{\partial \psi(\alpha^{SOC})}{\partial \alpha} \right]}_{-C_{ext}(\alpha^{SOC})} = 0$$

Or :

$$\frac{\partial W(\alpha)}{\partial \alpha} = \underbrace{\Delta(\alpha^{SOC})}_{\text{Social Value}} - \underbrace{[C + C_{ext}(\alpha^{SOC})]}_{\text{Social Cost of Fast Trading}} = 0$$

where $\Delta(\alpha^{SOC}) = \phi(\alpha^{SOC}) - \psi(\alpha^{SOC})$

Excessive Fast Trading

- **Result :** If $\sigma > 0$, the socially optimal level of fast trading is always smaller than the equilibrium level of fast trading, with a strict inequality when this level is strictly positive.
- **Intuition :**
 1. In making their investment decision, fast institutions compare the relative value of being fast ($\Delta(\alpha^{SOC})$) with the private cost of being fast (C) but they ignore the negative externality they impose on others ($C_{ext}(\alpha^{SOC})$).
 2. "The social planner" (regulator) accounts for this cost.
 3. This cost exists only because an increase in the level of fast trading raises adverse selection costs \rightarrow hence the condition $\sigma > 0$.

Policy Responses

- **Banning fast trading ?**
- **Slow-friendly markets ?**
- **Taxes ?**

Banning Fast Trading

- **Not a good idea because excessive fast trading does not mean that the optimal level is zero.**
- In fact, the socially optimal level of fast trading is strictly positive if :

$$\Delta(0) - C - C_{\text{ext}}(0) > 0. \quad (1)$$

- **Intuition :** The fast trading technology reduces “search costs” in fragmented markets.
- In fact Condition (1) is equivalent to :

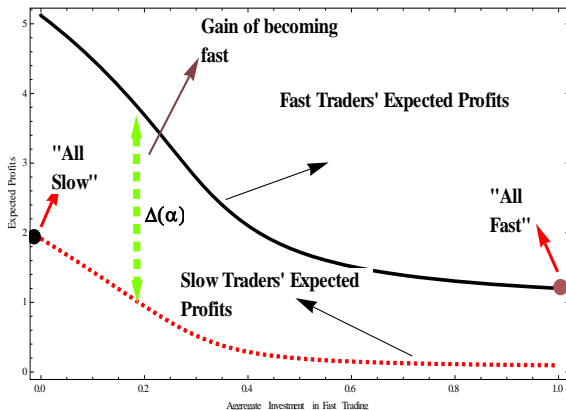
$$\underbrace{(1 - \lambda)E(|\delta|)}_{\text{Search Value}} + \underbrace{(2G(\sigma) - 1)(\sigma - E(|\delta| \mid |\delta| \leq \sigma))}_{\text{Speculative Value}} - C - \underbrace{(2G(\sigma) - 1)\sigma}_{\text{Externality Cost}} > 0. \quad (2)$$

- \implies Some fast trading is socially optimal if

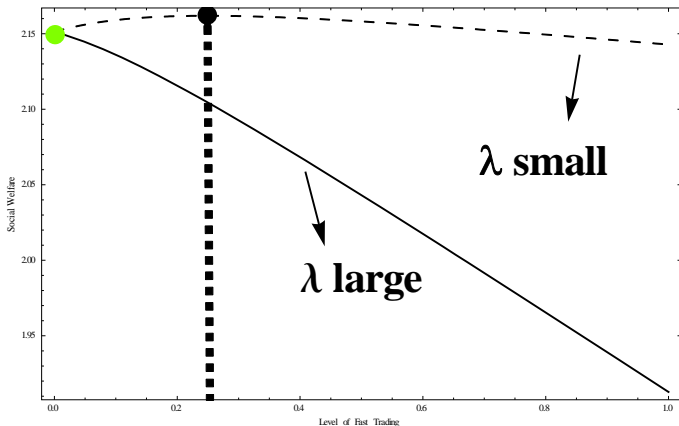
$$\lambda \leq \hat{\lambda}(\sigma, C, \pi)$$

where $\hat{\lambda}(\sigma, C, \pi)$ decreases in C , $\hat{\pi}$, and σ and is smaller than 1.

Example : Fast and Slow Traders' Expected Profits



Banning Fast Trading



Separating Slow and Fast



Source : Investors' Exchange (IEX).

Slow markets

- **Two market segments coexist :**
 1. **The fast segment :** markets in this segment operate as previously described
 2. **The slow segment (OTC, dark pools etc.) :** if an institution trades in this market segment, then it trades slow (does not get or cannot use information on θ_τ and is able to carry out its trade with proba λ only in each trading round).
- α : the fraction of institutions choosing to be fast
- β : the fraction of slow institutions choosing to trade in the slow market segment.

Equilibrium Spreads with Slow Markets

- As β increases \Rightarrow fewer slow institutions in the fast market segment \Rightarrow dealers have a higher exposure to adverse selection.
- **Implications :**
 1. The equilibrium bid-ask spread (illiquidity) in the fast market increases with β .
 2. Migration of slow traders to the slow market exerts a negative externality on traders remaining in the fast market.
- No adverse selection in the slow market \Rightarrow Zero spread in this market.

Equilibrium Investment Decisions with the Slow Market

- **Consider a slow institution :**

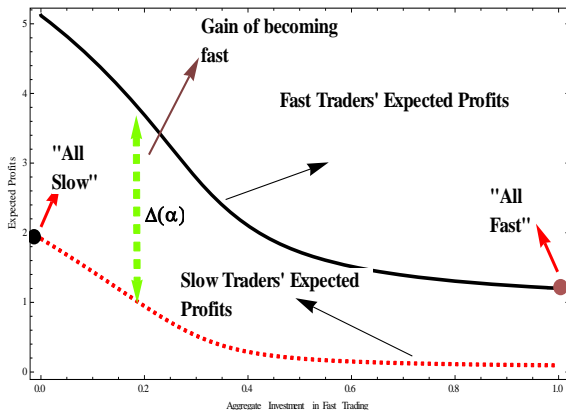
1. If it trades on the slow market it gets an expected profit : $\psi(0)$.
2. If it trades on the fast market, its expected profit is necessarily smaller since (i) it might not trade and (ii) if it trades, it pays the bid-ask spread.
3. As \implies All institutions who choose the be slow migrate to the slow market.

- Only fast institutions trade in the fast market \implies They obtain an expected profit equal to $\phi(1)$.

- **Implication : only two possible equilibria when slow and fast markets coexist**

1. $\phi(1) - \psi(0) > C$: All institutions are fast ($\alpha^* = 1, \beta^* = 0$)
2. $\phi(1) - \psi(0) < C$: All institutions are slow. ($\alpha^* = 0, \beta^* = 1$)

Example : Fast and Slow Traders' Expected Profits



Equilibrium Investment Decisions with the Slow Market

“Investors also have said that they have moved more of their trading into the dark because they have grown more distrustful of the big exchanges like the NYSE and the Nasdaq. Those exchanges have been hit by technological mishaps and become dominated by so-called high-frequency traders.” (“As markets heat up, trading slips into shadows,” New-York Times, 2013)

- Consistent with previous analysis, this evolution might also be responsible for the drop in fast trading profitability in recent years.

Slow Markets and Welfare

- Fix α and β .
- **Utilitarian Welfare with a slow market is :**

$$W(\alpha, \beta) = \underbrace{\beta(1 - \alpha)\psi(0)}_{\text{Welfare Slow on Slow Mkt}} + \underbrace{(1 - \beta)(1 - \alpha)\psi(\alpha^F)}_{\text{Welfare Slow on Fast Mkt}} + \underbrace{\alpha(\phi(\alpha^F) - C)}_{\text{Welfare Fast}}, \quad (3)$$

where $\alpha^F = \frac{\alpha}{\alpha + (1 - \beta)(1 - \alpha)}$.

- **Result :** In equilibrium, social welfare with a slow market and a fast market is always greater than with only a fast market :
 $W(0, 1) \geq W(\alpha^*, 0)$.

The slow markets approach is inefficient as well

- **PROBLEM** : The slow market results in no investment in fast trading even when this would be socially optimal.
- Hence, if $\lambda \leq \hat{\lambda}(\sigma, C, \pi)$, all trading on the slow market does **not** maximize social welfare
- **Intuition** : Institutions moving to the slow market exerts a negative externality on those staying on the fast market \implies Excessive slow trading \iff **Underinvestment in the fast trading technology.**

Pigovian Taxation

- **Taxation of investment in fast trading technologies is a more efficient approach.**
- **Consider the following :**
 1. A tax T_F^* paid by institutions investing in the fast technology.
 2. A tax T_S^* paid by institutions trading on the slow market ;
- The taxation scheme that maximizes social welfare is :
 1. $T_S^* > \psi(0)$: hence no investor joins the slow market
 2. $T_F^* = C_{ext}(\alpha^{SOC}) \Rightarrow$ exactly a fraction α^{SO} of institutions becomes fast.
- **Fairness** : Proceeds from the tax can be redistributed so that all institutions obtain the same welfare after redistribution.

Numerical Example

- suppose that institutions' private valuations are normally distributed with mean zero and s.d= 10 and $\sigma = 5$, $\pi = 0.9$, $r = 0$, and $\lambda = 0.0356$. The cost of the fast technology is $C = 4.77$.

Market Structure	α^*	Tax	Social Welfare
Fast Market only	1	0	2.14
Fast and Slow Markets	0	0	2.15
Pigovian Taxation	25%	$T^S > 2.15, T^F = 2.03$	2.16

Conclusions

- **Investment in fast trading technology is likely to be excessive :**
 1. Fast access to information \Rightarrow higher adverse selection costs \Rightarrow less scope for efficient realizations of gains from trade.
 2. Investment decisions can be complements \Rightarrow arms' race.
- **How to restore efficiency ?**
 1. Banning fast trading : too heavy-handed
 2. Slow markets : underinvestment can be an issue.
 3. Taxing investment in fast trading technology is more likely to be efficient.