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Reputation-based pricing and price improvements

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Abstract

Dealers often offer price improvements, relative to posted quotes, to their clients. In this paper, we propose an explanation to this practice. We also analyze its effects on market liquidity and traders' welfare. Enduring relationships allow dealers to avoid informed trades by offering price improvements to clients who do not trade with the dealer when they are informed. A dealer never observes whether a specific client is informed or not but he can avoid informed orders by conditioning his offers on past trading profits. Cream-skimming of uninformed order-flow increases the risk of informed trading for dealers without a relationship. Thus, authorizing price improvements increases bid-ask spreads and impairs the welfare of investors without a relationship. It may even decrease the welfare of investors who develop a relationship as they sometimes need to trade at posted quotes. The model predicts a positive relationship between (a) the price improvements granted to a specific investor and past trading profits with this investor or (b) the frequency of price improvements and bid-ask spreads.
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1. Introduction

Increasingly, securities trade in hybrid market structures where multiple trading venues coexist for the same asset. In particular, investors can often choose to execute their orders anonymously at posted quotes or to negotiate one-on-one a price improvement relative to these quotes. The organization of trading for Nasdaq stocks constitutes a good illustration. For these stocks, institu-

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tional investors can trade anonymously in ECNs such as Inet and Archipelago or they can seek to obtain a price improvement from a Nasdaq dealer. Barclay, Hendershott, and McCormick (2003) note that (pp. 2638–2639):

> ECNs compete with market-makers by providing anonymous and immediate execution [. . .] Market-makers, in contrast, enter into long-term agreements with brokers who preference or internalize their customers' orders regardless of the current quotes [. . .] Market-makers can negotiate directly with institutional traders and price discriminate based on their inferences about traders' motives and information. Because these negotiations are repeated, market-makers can quote competitive prices for large orders and impose costs on traders who take advantage of their private information.

This standard explanation for price improvements has been formalized by Seppi (1990) and Benveniste, Marcus, and Wilhelm (1992).[1] In these papers, investors commit not to trade with their relationship dealer when they have private information (Seppi, 1990) or they agree to truthfully disclose their trading motivation (Benveniste et al., 1992). In turn, relationship dealers offer attractive prices to uninformed investors. Enduring relationships are key for the enforcement of these no-informed trading agreements. Investors are deterred from breaching them by the threat of penalties on *future* trades when cheating is detected.[2]

Seppi (1990) and Benveniste et al. (1992) provide very important insights on the role of relationships and non-anonymity in financial markets. However, they leave some questions open. First, they assume that violations of no-informed trading agreements can be observed ex-post (i.e. after a transaction). In reality, trading motivations are never observed.[3] Then, one may wonder whether no-informed trading agreements can still be sustained when their violation cannot be observed. Second, the effect of no-informed trading agreements on traders' welfare has not been fully analyzed. Benveniste et al. (1992) consider the case in which liquidity is provided by a *single* dealer and in this case, they show that no-informed trading agreements are Pareto improving. However, in reality, several trading venues for the same security often coexist. It is unclear whether the conclusions obtained in Benveniste et al. (1992) apply in this case as well.

We address these issues. In our model, risk-averse investors have access to two different pools of liquidity : (i) a relationship dealer and (ii) "once-off dealers" who do not engage in long-term relationships. In each period, investors trade either to hedge endowment shocks or to exploit private information on the security's payoff. An investor implicitly agrees to not contact the relationship dealer (i.e. she trades only in the once-off dealer market) when she has private information. In turn, the relationship dealer sometimes (but not always) grants a price improvement to the investor. The relationship dealer is unable (at any point in time) to observe whether a given customer honors or not the terms of this no-informed trading agreement. We study whether he can enforce this agreement by granting price improvements contingent on his past trading profits with the investor. For simplicity, we focus on one specific profit-based pricing strategy. Namely, the dealer assigns a reputational index to each customer. This index increases each time the dealer books a loss after a transaction. The price improvement is offered if the value of the index is smaller than a threshold.

[1] Price improvements are a pervasive phenomenon, which have been documented in different markets: NYSE (Fialkowski & Petersen, 1994; Angel, 1997), Nasdaq (Huang & Stoll, 1996), London Stock Exchange (e.g. Reiss & Werner, 1996; Hansch, Naik, & Viswanathan, 1999) and Frankfurt Stock Exchange (Theissen, 2000).

[2] See Benveniste et al. (1992) for examples of sanctions that can be used by the NYSE specialist.

[3] Benveniste et al. (1992) assume that the relationship dealer observes a broker's true trading motivation with some probability $\Phi > 0$. Our point is that in reality $\Phi = 0$.

This pricing policy confronts an investor with the following dilemma when she possesses private information. She can earn a large return by exploiting her information at the dealer's expense. But the dealer records a loss and the investor's reputation is impaired (the index increases). Thus, informed trading increases the likelihood that the investor will be denied price improvements in the future. This is a concern to the investor because price improvements allow more efficient hedging when she is hit by liquidity shocks. For this reason, a loss in reputation is costly. For some values of the parameters, the reputational cost is larger than the immediate gain from informed trading. In these cases, we show that the implicit contract between the investor and her regular dealer is self-enforcing, despite the fact that the dealer cannot observe whether the investor abides to the terms of the contract or not.

As in the extant literature (Seppi, 1990 or Benveniste et al., 1992), price improvements play a key role: the investor honors the contract because she does not want to lose price improvements. However, as we explicitly model the mechanism enforcing the no-informed trading agreement, we obtain new predictions. In particular, a dealer's decision to grant a price improvement to a given customer depends on the profitability of his past transactions with this customer. Moreover, the model predicts a positive relationship between the frequency of price improvements and the bid-ask spread posted by once-off dealers.

We compare investors' welfare when the relationship market exists and when it does not. We distinguish two groups of investors : (a) investors who can establish a relationship and (b) investors who cannot establish a relationship. We also distinguish two distinct situations : (i) the case in which the once-off dealer market is viable in absence of the relationship market and (ii) the case in which it is not (informational asymmetries can prevent the once-off dealer market to operate). We find that the introduction of the relationship market is a Pareto improvement *only when* the once-off dealer market *is not* viable. Otherwise, it always harms the welfare of the investors without a relationship and, more surprisingly, it may even decrease the welfare of the investors who can establish a relationship.

The intuition for these findings is as follows. The relationship dealer cream-skims uninformed orders. This effect increases the fraction of informed orders routed to the once-off dealer market and impairs the liquidity of this market. For this reason, introduction of a relationship market imposes a welfare loss on investors who cannot forge long-term relationships. Investors with a relationship benefit from price improvements. However, they are also hurt by the decline in the liquidity of the once-off dealer market *because they sometimes need to trade in this market* (when they are informed or when they are denied price improvements). In some cases, this effect outweighs the welfare benefits associated with price improvements and *all* investors (including those with an access to the relationship market) suffer a welfare loss when the relationship market is introduced. When the once-off dealer market is not viable, the relationship market facilitates trades that, otherwise, would not occur. This improves the welfare of the investors who can establish a relationship and leaves unchanged the welfare of the investors without a relationship.

These results have implications for the design of marketplaces. Market structures (typically non-anonymous markets) that facilitate the emergence of long-term relationships are usually viewed as beneficial because they mitigate asymmetric information. Several authors however have suggested that technologies allowing a group of dealers to cream-skim uninformed orders might impair the liquidity of other trading venues and result in welfare losses (see Easley, Kiefer, & O'Hara, 1996 for instance). Our findings shed light on this question, as we delineate the conditions under which introduction of a relationship dealer improves or impairs welfare.

Rhodes-Kropf (in press) and Bernhardt, Dvoracek, Hughson, and Werner (2005) provide theories of price improvements that do not rely on asymmetric information. Rhodes-Kropf (in press) consider a static model in which investors have the possibility to negotiate price improvements. In his theory, price improvements to a specific customer are determined by the customer's bargaining power. Past profits earned by the dealer with this customer plays no role. Bernhardt et al. (2005) consider a repeated setting.[4] They show that a dealer should optimally grant larger price concessions to more valued customers. In line with this prediction, they find empirically that price improvements increase with past volume, a variable which determines the value of a relationship between a dealer and a broker. They also find that price improvements are positively related to past trading profits, as predicted by our model.

The paper is organized as follows. In the next section, we describe the model. Section 3 analyze the equilibria of the once-off dealer market, taking as given the proportion of informed orders submitted to this market. Section 4 studies the conditions under which a no-informed trading agreement can be sustained. Section 5 derives the implications of the model. Section 6 discusses robustness issues and Section 7 concludes. The proofs are collected in the Appendix A.

2. The model

We consider the market for a risky security. The model features (i) a risk averse investor, (ii) one risk neutral dealer (the relationship dealer) with whom some investors can engage in enduring relationships and (iii) many risk-neutral dealers ("once-off dealers") who cannot establish enduring relationships. In the rest of this section, we describe the choices faced by these different type of traders and the organization of the trading process.

2.1. The investor

The investor has trading opportunities at dates $1, 2, \ldots, t, \ldots \infty$. A 'period' is the time elapsed between opportunities and lasts δ units of times (e.g. 10 days). Thus, δ determines the trading frequency for the representative investor.

The risky security can be thought of as a derivative contract. In the middle of each period, the security pays $\tilde{\epsilon}$ where $\epsilon = +1$ or $\epsilon = -1$ with equal probabilities ($E(\tilde{\epsilon}) = 0$). The investor can also invest in a riskless asset. The (intra period) risk free rate is set to zero, for simplicity.

In each period, there are two types of trading opportunities. With probability $0 < \alpha < 1$, the investor receives perfect information on the payoff of the security in this period. With probability $(1 - \alpha)$, she receives a risky endowment and she has no privileged information on the security. At the end of each period, the risky endowment returns $\tilde{z} = \tilde{h}\tilde{\epsilon}$, where $\tilde{h} = +Q$ or $\tilde{h} = -Q$ with equal probabilities. Furthermore, $\tilde{h}$ and $\tilde{\epsilon}$ are independent. If h is positive (resp. negative), we will say that the investor has a *long* (resp. *short*) position. When she has a long (resp. short) position, the investor wants to sell (resp. buy) the security in order to reduce her risk exposure. She is perfectly hedged if she trades Q shares. The investor privately learns the direction of her hedging need (h) at the beginning of each period. After learning the direction of the hedging need or receiving information, the investor trades (or not) the security.

The investor starts each period with a constant endowment W_0 in the riskless asset. For tractability, we assume that she entirely consumes her wealth at the end of each period (no savings from

[4] Aitken, Garvey, and Swan (1995) also explicitly model long-term relationships between a broker-dealer and an investor. In their model, the broker-dealer trades at a loss with a client when the latter is informed and recoups this loss by charging large commissions when the client is uninformed.

one period to the next). Without affecting the results, we normalize W_0 to zero. As in Dow (1998), the investor's *per period* utility function is

$$U(W) = \gamma W \quad \text{for} \quad W > 0, \quad \text{and} \quad U(W) = W \quad \text{for} \quad W \leq 0, \tag{1}$$

where W is the investor's end of period wealth and $\gamma \in (0, 1)$. Notice that the *lower* is γ, the *larger* is the investor's risk aversion. Our results require the investor to be risk averse. The piecewise linear specification for the investor's per period utility yields simple explicit solutions.

To sum up, the investor can trade either to hedge or to benefit from her private information. More precisely, in every period, the investor has one of four possible types (trading motives):

(1) The investor needs to hedge a long position ($h = +1$).
(2) The investor needs to hedge a short position ($h = -1$).
(3) Informed with bad news ($\epsilon = -1$).
(4) Informed with good news ($\epsilon = +1$).

We denote each possible trading motive by θ_j, $j \in \{\text{L, SH, B, G}\}$. When she has trading motive θ_{L}, the investor has a Long position, etc.... Investors are not either always informed or always hedgers. Accordingly, in our model, the investor's trading motive *randomly* changes from one period to the next. We denote the investor's trading motive at date t by $\tilde{\theta}^t$.

2.2. Market structure : the relationship market and the once-off dealers market

In reality, investors often have access to multiple trading venues for a given stock. We mentioned the case of Nasdaq stocks in the introduction. The market structure for FTSE-100 stocks is another example. These stocks trade in two different systems : (i) an anonymous electronic limit order market (SETS) and (ii) a dealership market (see Naik & Yadav, 2004). In the dealership market, order execution is often arranged over the phone and investors have the possibility to negotiate a price improvement one-on-one. Thus, we assume that there are two trading venues for the risky security : (i) one market in which traders cannot develop long-term relationships (we call it the *once-off dealers market*) and (ii) one market in which traders have this possibility. In order to simplify the analysis, we assume that there is a single dealer acting in the relationship market (the *regular dealer*).

In reality, some investors cannot forge long-term relationships because, for instance, they trade too infrequently. Thus, we assume that the investor has a relationship with probability $\Psi \in [0, 1]$. In this case, she has access to *two* trading venues : (a) her regular dealer or (b) dealers with whom she has no established relationships (*once-off dealers*). Importantly, in each period, she can opt to trade *both* with once-off dealers *and* with the relationship dealer (whether informed or not). With probability $(1 - \Psi)$, the investor has no relationship and she trades only with once-off dealers.[5]

Trading in the once-off dealer market is anonymous.[6] Hence, when once-off dealers receive an order, they do not know (a) whether this order is submitted by a trader with a relationship or not

[5] Several authors (e.g. Benveniste et al., 1992) have pointed out that floor markets foster the development of long-term relationships. In this case, liquidity providers on the floor (specialists and floor brokers) are arguably able to identify traders with a relationship and those without a relationship. This corresponds to the case in which $\Psi = 1$ in our model.

[6] This is a natural assumption if one interprets the once-off dealer market as a stylised representation of an ECN. Alternatively, once-off dealers can simply be seen as dealers with whom the investor has no ongoing relationships. In this case, the investor can use a broker to preserve anonymity. The working of the interdealer market in the FX market is a good illustration. In this market, traders seeking immediacy can either negotiate execution prices one-on-one or trade anonymously through a broker (see Lyons, 1995). Several papers investigate the role of anonymity in financial markets.

(except if $\Psi = 1$) and (b) the investor's trading history. We also assume that the orders routed to one market by a specific investor are not observed by the dealer(s) operating in the other market.[7] Thus, the relationship dealer cannot constrain the investor to trade in only one trading venue (as in Seppi, 1990). More importantly, the regular dealer is unable to observe the investor's current or past trading motives, at any point in time and statements regarding this motive cannot be verified. This precludes pricing policies contingent on the investor's true trading motive (as in Benveniste et al., 1992).

Last, we assume that there is an exogenous probability $(1-\rho)$, *per unit of time*, that the investor and her relationship dealer ceases their relationship.[8] Thus, at the end of a period, there is a probability $\beta(\delta) = \rho^{\delta}$ that the relationship between the investor and her regular dealer will still be active at the beginning of the next period. Observe that $\beta(\delta)$ increases when the investor's trading frequency gets larger (δ decreases).

2.3. Trading process

In every period, we model the trading process as a two stage game. In the first stage, the investor observes her type and chooses the size of her trades with (a) her regular dealer and (b) the once-off dealers. We denote by q^{c} the order routed to the regular dealer and by q^{nc} the order routed to the once-off dealers market.[9] A positive (negative) quantity indicates an order to buy (sell). The investor's trading strategy, $q(\theta, S) = (q^{\mathrm{c}}(\theta, S), q^{\mathrm{nc}}(\theta, S))$, depends on her type and a state variable (S), which is determined by her trading history with the regular dealer (see Section 4).

If the regular dealer is contacted by the investor ($q^{\mathrm{c}} \neq 0$), he chooses the price at which he accommodates the investor's order. We refer to $p^{\mathrm{c}}(q, S)$ as the regular dealer's bidding strategy. It gives the price offered by the dealer when (a) he receives a request to execute an order of size q and (b) the value of the state variable is S. The dealer's offer depends on his belief regarding the payoff of the security. We denote by $\phi^{\mathrm{c}}(q, S)$ the probability distribution of this payoff conditional on the order size and the state variable. The regular dealer's expected profit when he trades q shares is

$$\pi(p^{\mathrm{c}}(q, S), q) = q[p^{\mathrm{c}}(q, S) - E_{\phi^{\mathrm{c}}}(\tilde{\epsilon})]. \tag{2}$$

The trading process with once-off dealers follows the same steps. However, once-off dealers' bidding strategies are *not* contingent on the investor's trading history as they do not observe it. Their posterior belief conditional on the investor's trade size is denoted $\phi^{\mathrm{nc}}(q^{\mathrm{nc}})$. The investor's order for once-off dealers is eventually routed to the dealer who posts the best price. If several dealers post the best price, one is randomly chosen to execute the order. Under these assumptions, competition leads once-off dealers to post a price equal to their valuation for the risky security

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These include Forster and George (1992), Theissen (2000), Garfinkel and Nimalendran (2002), Simaan, Weaver and Withcomb (2003) or Foucault, Moinas, and Theissen (2005).

[7] This is a reasonable assumption as trade execution reports do not usually reveal the identities of contra-side parties. For instance, for trades taking place on ECNs, the ECN is listed as the contra-side party to maintain post-trade anonymity.

[8] There are various exogenous events which could lead to a termination of the relationship between the investor and the regular dealer. Relocation of the trading post for a given stock listed on the NYSE is one example. In an interesting natural experiment, Battalio, Ellul, and Jennings (2005) show that this relocation often terminates the relationships between the relocated specialist and a specific crowd of floor brokers.

[9] Superscripts 'c' and 'nc' stand for 'cooperative' and 'non-cooperative', respectively, for reasons which will become clear below.

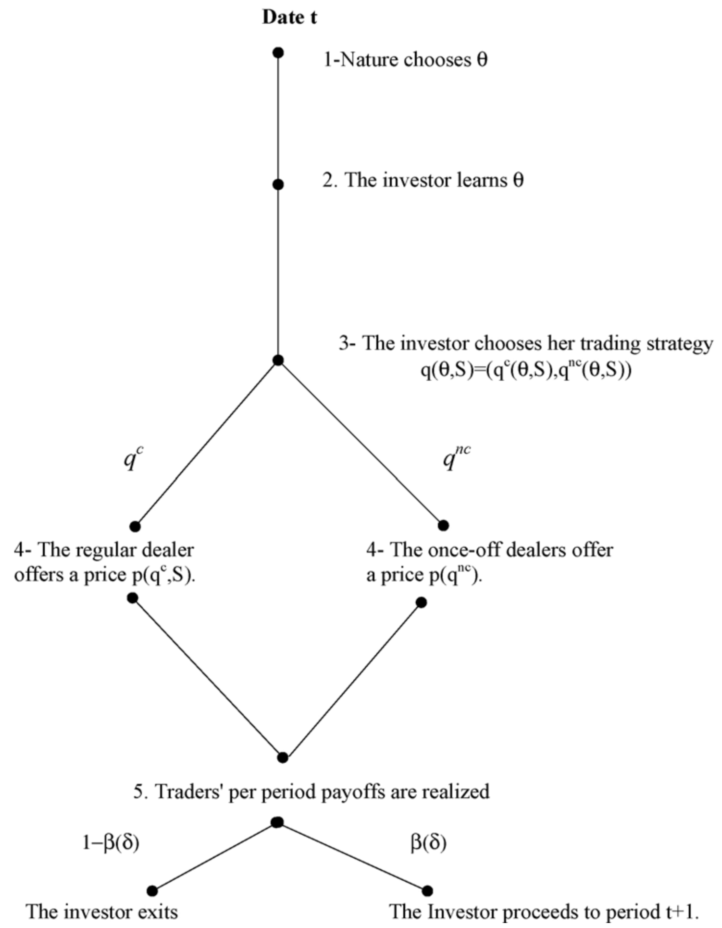

Fig. 1. Sequence of events.

(as in Glosten and Milgrom, 1985 for instance), i.e.

$$p^{\mathrm{nc}}(q) = E_{\phi^{\mathrm{nc}}}(\tilde{\epsilon} \mid q^{\mathrm{nc}} = q). \tag{3}$$

Fig. 1 summarizes the sequence of events in each period.

3. Equilibria in the once-off dealers market

In this section, we analyze the equilibria that arise in the once-off dealer market. We denote by λ the probability that an order routed to the once-off dealer market comes from an informed investor. We show that there are two types of equilibria in the once-off dealer market : (i) either the once-off dealer market is viable and in this case once-off dealers execute buy (resp. sell) orders at a markup (resp. discount) equal to $s^{\mathrm{nc}} = \lambda$ or (ii) the once-dealer market is not viable, i.e. no trading takes place in this market.

In Section 4.3, we will derive λ endogenously and we will show that in general $\lambda \geq \alpha$. However, for the moment, we take λ as given and we analyze the Perfect Bayesian Equilibria (PBE) of the trading process in the once-off dealer market.[10] A PBE of the once-off dealers' market is a set $(q^{nc*}(.), p^{nc*}(.), \phi^{nc*}(.))$ such that

(1) The trading strategy $q^{nc*}(.)$ maximizes the investor's expected utility given the once-off dealer's bidding strategy, $p^{nc*}(.)$.
(2) The bidding strategy $p^{nc*}(.)$ is such that $p^{nc*}(q) = E_{\phi^{nc*}}(\tilde{\epsilon} \mid q)$.
(3) The dealers' posterior belief $\phi^{nc*}(.)$ is derived from the investor's trading strategy using Bayes rule where possible.

In order to derive the results of this section, we assume that the investor trades in only one market when she needs to hedge. We will show that this is indeed optimal in the next section. Thus, conditional on trading in the once-off dealer market, the investor's hedging demand is identical whether she has a relationship or not. The next lemma establishes some properties common to all equilibria.

Lemma 1. *All the equilibria in the once-off dealers' market have the following properties*:

- *The investor buys the same quantity when she has a short position or when she receives good news, i.e.* $q^{nc*}(\theta_{SH}) = q^{nc*}(\theta_G) \geq 0$.
- *The investor sells the same quantity when she has a long position or when she receives bad news, i.e.* $q^{nc*}(\theta_L) = q^{nc*}(\theta_B) \leq 0$.
- *In the equilibria for which* $q^{nc*}(\theta_{SH}) > 0$, *the once-off dealers charge a price equal to* $p^{nc*}[q^{nc*}(\theta_{SH})] = s^{nc}(\lambda)$ *when they receive the equilibrium buy order*. *In the equilibria for which* $q^{nc*}(\theta_L) < 0$ *the once-off dealers charge a price equal to* $p^{nc*}[q^{nc*}(\theta_L)] = -s^{nc}(\lambda)$ *when they receive the equilibrium sell order, with* $s^{nc}(\lambda) \stackrel{\text{def}}{=} \lambda$.

The investor sells the security for one of two possible reasons: (i) she needs to hedge a long position or (ii) she has received bad news. She sells the same quantity in these two cases in order to avoid detection by the dealers when she is informed. For the same reason, she buys the same quantity when she has a short position or when she receives good news. In order to break-even, once-off dealers must price the security at a discount (a markup) relative to the security unconditional expected value when the investor chooses to sell (buy) the security. This wedge, $s^{nc}(\lambda)$, is the bid-ask spread charged by the dealers. Observe that the bid-ask spread is the same in all equilibria in which there is trading between the investor and the once-off dealers.

Fig. 2 gives the investor's hedging demand when she needs to hedge a short position as a function of the spread ($s^{nc}(\lambda)$) charged by the once-off dealers.[11] Not surprisingly, the hedging demand decreases with the size of the spread and it becomes nil when the spread is larger than $\overline{s}(\gamma) = (1-\gamma)/(1+\gamma)$. Suppose for the moment that $s^{nc}(\lambda) < \overline{s}(\gamma)$, i.e. $\gamma < (1-\lambda)/(1+\lambda)$ and consider the following trade size:

$$\overline{q}(\lambda) \stackrel{\text{def}}{=} \frac{Q}{1+s^{nc}(\lambda)}. \tag{4}$$

[10] We consider equilibria in pure strategies only. Moreover we assume that the investor does not trade when she is indifferent between trading and not trading.
[11] The investor's optimal sell order when she has a long position is identical. Lemma 4 in the Appendix A formally derives the investor's hedging demand.

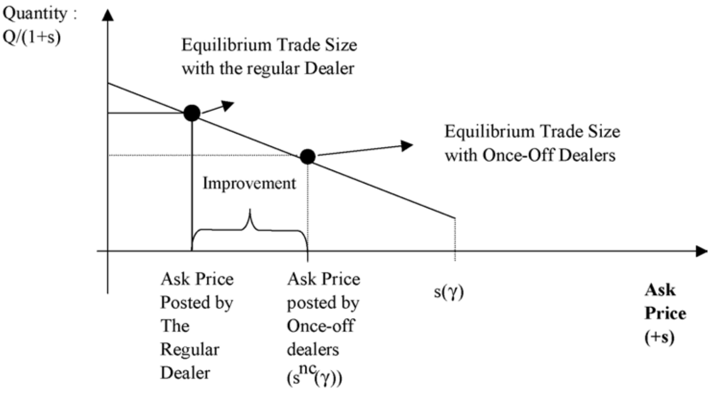

Fig. 2. The investor's hedging demand.

This is the investor's hedging demand when dealers charge a spread equal to $s^{\mathrm{nc}}(\lambda) < \bar{s}(\gamma)$ (see Fig. 2). The next proposition establishes that there is an equilibrium in which the investor trades $\bar{q}(\lambda)$ with the once-off dealers if $\gamma < (1-\lambda)/(1+\lambda)$.

Proposition 1. *When $\gamma < (1-\lambda)/(1+\lambda)$, there is an equilibrium in which the investor's trade size is $\bar{q}(\lambda)$ and once-off dealers' spread is equal to $s^{\mathrm{nc}}(\lambda)$. More precisely, in equilibrium*

(1) *The investor's trading strategy is* (a) $q^{\mathrm{nc}*}(\theta_{\mathrm{L}}) = q^{\mathrm{nc}*}(\theta_{\mathrm{B}}) = -\bar{q}(\lambda)$, (b) $q^{\mathrm{nc}*}(\theta_{\mathrm{SH}}) = q^{\mathrm{nc}*}(\theta_{\mathrm{G}}) = \bar{q}(\lambda)$,
(2) *and the once-off dealers' bidding strategy is*: (a) $p^{\mathrm{nc}*}(-\bar{q}) = -s^{\mathrm{nc}}(\lambda)$, (b) $p^{\mathrm{nc}*}(\bar{q}) = s^{\mathrm{nc}}(\lambda)$, (c) $p^{\mathrm{nc}*}(-q) = -1 \quad \forall q \neq \bar{q}, q > 0$, *and* (d) $p^{\mathrm{nc}*}(q) = +1 \quad \forall q \neq \bar{q}, q > 0$.[12]

Order sizes different from $\bar{q}(\lambda)$ can be supported in equilibrium.[13] The equilibrium described in Proposition 1 appears more sensible than other equilibria because it Pareto dominates any other equilibria. Actually, the investor's ex ante expected utility in this equilibrium is larger than her ex ante expected utility in any other equilibrium.[14] As for the once-off dealers, they are indifferent between all equilibria as they always obtain zero expected profits. Consequently, when $\gamma < (1-\lambda)/(1+\lambda)$, we focus on this equilibrium of the once-off dealers' market in the rest of the paper.

The spread, $s^{\mathrm{nc}}(\lambda)$, increases with once-off dealers' belief regarding the risk of trading with an informed investor, λ. When λ is so large that $s^{\mathrm{nc}}(\lambda) \geq \bar{s}(\gamma)$, i.e. $\gamma \geq (1-\lambda)/(1+\lambda)$, the once-off dealers market breaks down. Actually, once-off dealers must charge a bid-ask spread at least equal to $s^{\mathrm{nc}}(\lambda)$ in order to break-even. But for this spread, the investor's hedging demand is zero (see Fig. 2). Hence, trades can occur only when the investor is informed, which leads to a no-trade outcome. The corresponding equilibrium is described in the next proposition.

[12] Parts (c) and (d) specify the spread charged by the dealers for trade sizes which are not chosen in equilibrium, that is which are out-of-the equilibrium path. This is necessary for a complete description of the equilibrium.
[13] As usual in signalling games, the multiplicity of equilibria reflects the absence of restrictions imposed by the PBE concept on dealers' belief, $\phi^{\mathrm{nc}*}$, for trade sizes out-of-the equilibrium path.
[14] This claim is an immediate consequence of Lemma 4 in Appendix A. We skip its proof for brevity. It is available upon request.

Proposition 2. *When $\gamma \geq (1-\lambda)/(1+\lambda)$, the once-off dealer market is shut down: The unique equilibrium is such that* (i) *the once-off dealers sell* (*buy*) *at price* $p^{\mathrm{nc}*} = 1(-1)$ *and* (ii) *the investor never trades, i.e.* $q^{\mathrm{nc}*}(\theta) = 0, \forall\theta$.

Together, Propositions 1 and 2 imply that the condition $\gamma < (1-\lambda)/(1+\lambda)$ is necessary and sufficient for the viability of the once-off dealer market. Observe that the once-off dealer market is not viable when (i) the probability of informed trading (λ) is large or (ii) the investor's hedging need is weak because she is not very risk averse (γ large). The collapse of the once-off dealer market in these cases is not surprising given the adverse selection problem faced by the dealers. Other authors (e.g. Glosten, 1989) have developed models where market breakdowns occur under similar conditions.

Remark. Observe that once-off dealers' equilibrium prices are entirely determined by parameter λ. They do not directly depend on Ψ. We will show in Section 4.3 that once-off dealers' quotes indirectly depend on Ψ because λ is in part determined by Ψ. It is also worth stressing that the results obtained in this section also apply when no investor has a relationship (i.e. $\Psi = 0$). Of course, in this case, $\lambda = \alpha$.

4. Enforcing a no informed trading agreement

Now, we focus on the relationship between the investor and her regular dealer (i.e. we consider the behavior of an investor *with* a relationship). Our purpose is to analyze the conditions under which the following implicit contract (the *no-informed trading agreement*) is self-enforcing : (a) the investor contacts her regular dealer *only* when she needs to hedge and (b) in return, the regular dealer offers a price improvement to the investor.[15] We proceed as follows. First, in Section 4.1, we describe one specific pricing policy (" *the scoring policy*") for the relationship dealer. This policy makes violations of the no-informed trading agreement costly for the investor. Then, in Section 4.2, we derive a simple condition ("*the no-informed trading condition*") that a scoring policy must satisfy to discipline the investor. Finally (Section 4.3), we close the model by deriving the equilibrium value of λ for given characteristics of the no-informed trading agreement between the investor and the regular dealer. We then show that there is a non-empty set of parameters for which the no-informed trading agreement is self-enforcing.

It is worth stressing that, throughout, our analysis encompasses both (i) the case in which the once-off dealer market is viable ($\gamma < (1-\lambda)/(1+\lambda)$) and (ii) the case in which the once-off dealer market is not viable ($\gamma \geq (1-\lambda)/(1+\lambda)$).

4.1. No-informed trading agreements : a formal definition

In this section, we define formally the strategies followed by the regular dealer and the investor over the course of their relationship when they are bound by a no-informed trading agreement.

The relationship dealer. First, consider the pricing policy followed by the regular dealer. This policy is such that the relationship between the dealer and the investor alternates between cooperative phases and non-cooperative phases.

[15] It is worth stressing that we do not analyze the observation phase during which the investor and the regular dealer build up their relationship. Rather, we study the nature of this relationship in "cruise speed".

(1) In *cooperative phases*, the dealer charges a spread $s^{c} < \mathrm{Min}\{s^{nc}(\lambda), \overline{s}\}$ for a trade of size $q^{c}(s^{c}) \stackrel{\text{def}}{=} Q/(1+s^{c})$. This is the investor's optimal order when she is uninformed and the dealer's spread is s^{c} (Fig. 2). For other trade sizes, the dealer charges a price of $+1$ for buy orders and -1 for sell orders.
(2) After each encounter, the dealer assigns a 'score', $S \geq 0$, to the investor. This score depends on the profitability of his trades with the investor. When the dealer loses money, he increases the score by one unit. As $s^{c} < 1$, this happens when the investor sells the asset and subsequently the asset price decreases or when the investor buys the asset and subsequently the asset price increases. For all the other trades, the dealer earns his posted spread (if the asset payoff is zero) or even more. For these profitable trades or when there is no trade, the score is unchanged.
(3) When the score reaches the threshold, S^{*}, a non-cooperative phase begins.
(4) *In non-cooperative phases*, the regular dealer charges a price of +1 for buy orders and -1 for sell orders. This is as if he refused to trade with the investor since the investor never finds optimal to trade at these prices. In each period, he increases the score by 1 unit in order to keep track of the time elapsed since the beginning of the non-cooperative phase. When the score is equal to $S^{*} + T$, a new cooperative phase begins and the score is reset at zero. Thus, a non-cooperative phase lasts $T \geq 1$ periods.

We refer to this pricing policy as *a scoring policy* and we denote it $p^{lt}(T, S^{*}, s^{c})$ (superscript 'lt' stands for 'long-term'). A scoring policy is characterized by three parameters: (i) the '*trigger value*' of the score (S^{*}), (ii) the *length of the non cooperative phase* (T) and (iii) the *size of the spread during cooperative phases* (s^{c}). In every period, the dealer's pricing strategy is completely determined by the investor's score, i.e. the score serves as a state variable.[16] During cooperative phases, the relationship dealer offers *a price improvement* relative to the price posted by once-off dealers as $(s^{nc}(\lambda) - s^{c}) > 0$ (if the once-off dealer market is active). The relationship dealer's policy is such that he will not trade during non-cooperative phases. This is not crucial but this simplifies the presentation of the model. The important point is that the investor has no access to price improvements during these phases.

The scoring policy can be viewed as *an incentive scheme*. In order to see this point, observe that the investor can behave in two different ways: '*cooperatively*' (she contacts her regular dealer only when she is uninformed) or '*non-cooperatively*' (she trades with the regular dealer also when she is informed). Cooperative behavior is optimal for the investor only if she is penalized for behaving non-cooperatively. However, the dealer never observes the investor's true trading motivation. Thus, penalties must be based on variables correlated with the investor's unobserved trading motivation. This is the case for trading profits. To see this point, observe that the probability of a trading loss for the dealer, conditional on a trade taking place, is

$$\mathrm{Prob}^{c}(\mathrm{Loss}) = \frac{1}{2},$$

if the investor behaves cooperatively. If the investor behaves non-cooperatively, this probability is

$$\mathrm{Prob}^{nc}(\mathrm{Loss}) = \alpha + \frac{1-\alpha}{2} = \frac{(1+\alpha)}{2},$$

[16] This is the reason why we assume that the score keeps increasing during non-cooperative phases. In this way, we can express the regular dealer's bidding strategy as a function of the score only (we do not have to label the bidding strategy differently during cooperative and non-cooperative phases).

Hence, the distribution of trading losses for the regular dealer depends on the investor's behavior. The likelihood of a trading loss is larger when she exploits her private information at the dealer's expense. The dealer must therefore penalize the investor when he books a loss. This is exactly what is achieved by a scoring policy. Actually, if the dealer bears a loss, he increases the investor's score and this shortens the expected duration of the cooperative phase as this phase stops when the score hits the threshold S^*. As shown in the next section, this is costly for the investor because hedging is more efficient when she receives price/size improvements.

We interpret the score as a *reputational index*.[17] The investor's reputation deteriorates as the score increases. The scoring policy is designed in such a way that the investor loses her reputational capital when she exploits her information at the expense of her regular dealer.

The investor. Now, we describe formally the investor's behavior when she adheres to the no-informed agreement. We define a *cooperative trading policy*, denoted $q^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$, as follows:

(1) *During cooperative phases*, the investor behaves cooperatively. That is, she contacts the relationship dealer when she is uninformed and she trades with the once-off dealers when she is informed. More formally, when her score is $S < S^*$, the investor's trading strategy is:

$$\begin{cases} q^*(\theta_{\mathrm{L}}, S) \stackrel{\mathrm{def}}{=} \left(-\dfrac{Q}{1+s^{\mathrm{c}}}, 0\right) \\ q^*(\theta_{\mathrm{SH}}, S) \stackrel{\mathrm{def}}{=} \left(\dfrac{Q}{1+s^{\mathrm{c}}}, 0\right) \\ q^*(\theta, S) \stackrel{\mathrm{def}}{=} (0, q^{\mathrm{nc}*}(\theta)) \quad \text{for} \quad \theta \in \{\theta_{\mathrm{B}}, \theta_{\mathrm{G}}\} \end{cases} \tag{5}$$

where $q^{\mathrm{nc}*}(.)$ is defined as in Section 3. The first (resp. second) entry in $q^*(., .)$ describes the order sent to the regular dealer (resp. once-off dealers).

(2) *During non-cooperative phases*, the investor only trades with once-off dealers. Her trading strategy is therefore :

$$q^*(\theta, S) \stackrel{\mathrm{def}}{=} (0, q^{\mathrm{nc}*}(\theta)) \quad \forall \theta \quad \text{and} \quad S > S^*. \tag{6}$$

Recall that if $\gamma \geq (1-\lambda)/(1+\lambda)$ then the once-off dealers' market is shut down. In this case $q^{\mathrm{nc}*} = 0$ and the investor does not trade during non-cooperative phases.

4.2. Cooperative trading

In this section, we derive a simple condition (Proposition 3) under which the investor does not violate the no-informed trading agreement when the relationship dealer uses a scoring policy. This condition guarantees that the investor is better off *not* abusing her reputation at any stage of the relationship. We also derive the conditions under which the relationship dealer has not incentive to renege on the terms of the scoring policy (Proposition 4) when the investor follows the cooperative trading policy.

[17] This interpretation is valid for cooperative phases only. During non-cooperative phases, the score is just a measure of the time elapsed since the beginning of the phase.

When the investor follows the policy $q^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$, her long-run *expected payoff* at date τ when her score is S writes :

$$V(\theta^\tau, S) = E[U(q^*(\theta^\tau, S))] + \sum_{t=\tau+1}^{+\infty} \beta^{t-\tau} E[U(q^*(\tilde{\theta}^t, \tilde{S}_t))|S_\tau = S]. \tag{7}$$

Let $V(in, S)$ be the investor's expected payoff conditional on the investor being informed and $V(he, S)$ be the investor's expected payoff when she has a hedging need, conditional on the score being equal to S.[18] The investor's ex ante expected payoff when she follows the cooperative trading policy is

$$V(S) = \alpha V(in, S) + (1-\alpha) V(he, S). \tag{8}$$

The next lemma shows that the value of the relationship for the investor (i.e. $V(.)$) decreases with her score *during the cooperative phase*. This result formalizes the idea that a loss in reputation is costly for the investor. Let $\Delta V(S) \stackrel{\mathrm{def}}{=} V(S) - V(S+1)$ be the change in the value of the relationship when the score increases by one unit. We refer to this variable as the *reputational cost*.

Lemma 2. (*A loss in reputation is costly*) *The value of the relationship,* $V(.)$, *decreases with* S *for* $S \leq S^*$, *i.e.* $\Delta V(S) > 0$. *Furthermore, the reputational cost increases with* S. *Thus,* $\Delta V(S) > \Delta V(0)$ *for* $1 < S \leq S^* - 1$.

The first part of the lemma is quite intuitive. The larger is the score, the shorter is the expected length of the cooperative phase. This effect reduces the value of her relationship for the investor as hedging is less efficient during non cooperative phases. The second part of the lemma indicates that a loss in reputation is the least costly when the investor's reputational capital is maximal (the score is equal to zero).

We can now analyze the conditions under which the investor is better off not abusing the regular dealer *at any stage of the relationship*. To this end, consider the case in which the investor is informed and her relationship with the dealer is in a cooperative phase ($S < S^*$). Independently of her behavior in the relationship market, the investor can trade in the once-off dealer market (if the once-off dealer market is open). Let $\bar{U}_{in}^{\mathrm{nc}}$ be the investor's corresponding expected utility.[19] If, in addition, she contacts the relationship dealer and masquerades as being uninformed, she can obtain an additional expected utility equal to $\bar{U}_{in}^{\mathrm{c}}(s^{\mathrm{c}}) \stackrel{\mathrm{def}}{=} (\gamma(1-s^{\mathrm{c}})Q)/(1+s^{\mathrm{c}})$ (as the regular dealer's spread is s^{c} for a trade size equal to $Q/(1+s^{\mathrm{c}})$ shares). However, following this transaction her reputation deteriorates (her score is increased by one unit) and the continuation value for the relationship becomes $\beta V(S+1)$. Overall, the investor's total expected payoff if she violates the no-informed trading agreement is

$$\frac{\gamma(1-s^{\mathrm{c}})Q}{1+s^{\mathrm{c}}} + \bar{U}_{in}^{\mathrm{nc}} + \beta V(S+1).$$

If the investor abstains from contacting the relationship dealer, she bears *the opportunity cost* of not exploiting her information against the relationship dealer but she does not impair her reputation. In this case, she obtains an expected payoff equal to

$$\bar{U}_{in}^{\mathrm{nc}} + \beta V(S).$$

[18] Formally $V(in, S) = \frac{1}{2}V(\theta_{\mathrm{B}}, S) + \frac{1}{2}V(\theta_{\mathrm{G}}, S)$ and $V(he, S) = \frac{1}{2}V(\theta_{\mathrm{L}}, S) + \frac{1}{2}V(\theta_{\mathrm{SH}}, S)$.

[19] It is straightforward that $\bar{U}_{in}^{\mathrm{nc}} = (\gamma(1-s^{\mathrm{nc}})Q)/(1+s^{\mathrm{nc}})$ if the once-off dealer market is active. Otherwise, the investor does not trade in the once-off dealer market and obviously $\bar{U}_{in}^{\mathrm{nc}} = 0$.

We deduce that the investor optimally chooses *not* to contact the regular dealer when she is informed *iff*:

$$\bar{U}_{in}^{\mathrm{nc}} + \beta V(S) \geq \bar{U}_{in}^{\mathrm{c}}(s^{\mathrm{c}}) + \bar{U}_{in}^{\mathrm{nc}} + \beta V(S+1) \quad \forall S \leq S^* - 1.$$

That is,

$$\beta(\Delta V(S)) \geq \bar{U}_{in}^{\mathrm{c}}(s^{\mathrm{c}}) \quad \forall S \leq S^* - 1. \tag{9}$$

The L.H.S. of this inequality is the reputational cost borne by the investor if she trades with her relationship dealer when she is informed. The R.H.S. is the utility gain that she obtains in this case. In line with intuition, the investor does not exploit her information against her regular dealer if the reputational cost is larger than the immediate utility gain. Notice that Condition (9) must be satisfied for all values of the score, that is even when the investor's reputational capital is maximal ($S = 0$).

The last part of Lemma 2 implies that Condition (9) is equivalent to :

$$\beta(\Delta V(0)) \geq \bar{U}_{in}^{\mathrm{c}}(s^{\mathrm{c}}). \tag{10}$$

Clearly, this condition cannot be satisfied when β is too small. Recall that β increases with trading frequency ($\beta = \rho^{\delta}$). Thus, traders who do not trade frequently enough cannot be disciplined. They will always be denied price improvements and will only trade in the once-off dealer market. Interestingly, this might encourage an investor to trade more in order to get a reputation or to delegate her trading to agents (portfolio managers, brokers) who trade very frequently.

From now on, in order to reduce the number of parameters, we focus on the limit case in which the investor's β goes to 1. This is the case when (i) the likelihood (ρ) of an exogenous termination of the relationship between the regular dealer and the investor is very small or (ii) the investor's trading frequency is large (δ very small). In this case, solving for the value function $V(.)$, we can rewrite the previous condition in terms of the parameters of the scoring policy and the exogenous parameters (γ, α and Q). This yields the following result.

Proposition 3. *When the dealer follows the scoring policy* $p^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$, *the cooperative trading policy* $q^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$ *is optimal if and only if*

$$\frac{\Delta U(s^{\mathrm{c}}, \lambda)}{\bar{U}_{in}^{\mathrm{c}}(s^{\mathrm{c}})} \geq (\mathrm{Prob}(\Delta S = +1) + \Lambda), \tag{11}$$

with $\mathrm{Prob}(\Delta S = +1) \stackrel{\mathrm{def}}{=} (1-\alpha)/2$, $\Lambda \stackrel{\mathrm{def}}{=} S^*/T$, $\bar{U}_{in}^{\mathrm{c}}(s^{\mathrm{c}}) \stackrel{\mathrm{def}}{=} (\gamma(1-s^{\mathrm{c}})Q/(1+s^{\mathrm{c}})$ *and*

$$\Delta U(s^{\mathrm{c}}, \lambda) \stackrel{\mathrm{def}}{=} \begin{cases} \dfrac{(\bar{s}(\gamma) - s^{\mathrm{c}})(1-\alpha)Q}{(1+\bar{s}(\gamma))(1+s^{\mathrm{c}})} & \text{if} \quad \gamma \geq \dfrac{1-\lambda}{1+\lambda}, \\[2ex] \dfrac{(s^{\mathrm{nc}}(\lambda) - s^{\mathrm{c}})(1-\alpha)Q}{(1+s^{\mathrm{nc}}(\lambda))(1+s^{\mathrm{c}})} & \text{if} \quad \gamma < \dfrac{1-\lambda}{1+\lambda}. \end{cases} \tag{12}$$

We refer to Condition (11) as the "no informed trading condition" . Actually, it is optimal for the investor to follow the cooperative trading policy, that is to honor the no informed trading agreement, *iff* this condition is satisfied. Variable $\Delta U(s^{\mathrm{c}}, \lambda)$ is the *per period* welfare gain brought about by cooperation for the investor (i.e. the difference between her per-period expected utility during cooperative phases and her per-period expected utility during non-cooperative phases). Thus, the no-informed trading condition states that the investor adopts a cooperative behavior iff the ratio of the per-period welfare gain brought up by cooperation to the per period gain from breaching the no-informed trading agreement ($\bar{U}_{in}^{\mathrm{c}}(s^{\mathrm{c}})$) is large enough.

We now consider the conditions under which the regular dealer is better off using the scoring policy $p^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$ when the investor follows the cooperative trading policy $q^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$. The orders routed to the regular dealer by the investor are "captive" as there are not exposed to multiple dealers competing for the execution of this order. The dealer might be tempted to exploit this feature by charging a larger spread than the promised spread, s^{c}. The investor has some leverage, however: she can threaten to terminate her relationship with the dealer if he reneges on their implicit agreement. This threat limits the market power of the relationship dealer, as pointed out by Bernhardt et al. (2005). In order to formalize this idea, we add the following provision to the implicit contract between the investor and the regular dealer. The dealer and the investor cooperate (he uses a scoring policy and she uses the associated cooperative policy) as long as the dealer follows the promised scoring policy. Otherwise, the investor stops contacting the regular dealer forever and the regular dealer charges a price equal to $+1$ (-1) for a buy (sell) order if he is contacted by the investor. This course of actions following a breach of the implicit contract by the dealer forms an equilibrium. In particular the specification of the dealer's quotes implies that implementing her threat is optimal for the investor.[20]

The regular dealer's long-run expected profit at date τ when he and the investor follow the cooperative policies $p^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$ and $q^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$ is

$$\Pi(S_\tau) = E[\pi(p^{\mathrm{c}*}(\tilde{q}_\tau, S_\tau), \tilde{q}_\tau)|S_\tau] + \sum_{t=\tau+1}^{+\infty} \beta^{t-\tau} E[\pi(p^{\mathrm{c}*}(\tilde{q}_t, \tilde{S}_t), \tilde{q}_t)|S_\tau], \tag{13}$$

where $p^{\mathrm{c}*}(.,.)$ is the regular dealer's bidding strategy in each period. The investor's threat obliges the dealer to balance the one-shot profit from charging a supra-normal spread against the value of his future profits with the investor. Consider the dealer in a cooperative phase when the investor's score is S. If he charges a spread equal to s^{c}, his total expected profit is

$$s^{\mathrm{c}} \frac{Q}{1+s^{\mathrm{c}}} + \beta \left[\frac{1}{2}\Pi(S+1) + \frac{1}{2}\Pi(S)\right] \quad \text{for} \quad S \leq S^* - 1.$$

If instead the dealer deviates and charges a spread equal to $s^{\mathrm{d}} > s^{\mathrm{c}}$, he obtains a total expected profit equal to $s^{\mathrm{d}}(Q/(1+s^{\mathrm{c}}))$ since from the next period onward, the investor will abstain from trading with the dealer. Hence, the dealer is better off charging the promised spread iff

$$s^{\mathrm{d}} \frac{Q}{1+s^{\mathrm{c}}} \leq s^{\mathrm{c}} \frac{Q}{1+s^{\mathrm{c}}} + \beta \left[\frac{1}{2}\Pi(S+1) + \frac{1}{2}\Pi(S)\right]. \tag{14}$$

Other things equal, the value of the future profits earned with the investor (the term inside the brackets) grows with β. When β goes to 1, this value become infinite *if* $s^{\mathrm{c}} > 0$ and $T < \infty$.[21] Since s^{d} is bounded (otherwise the investor would simply refuse to trade), the condition for no opportunistic behavior on the dealer's side is satisfied for β large enough. Thus, for β close to 1, we obtain the following result.

[20] Given the investor's strategy, the dealer does not expect to receive an order if he breaches the implicit contract. Thus, after a breach of the implicit contract, the dealer's beliefs (and therefore his pricing strategy) conditional on receiving an order can be chosen arbitrarily.

[21] When $s^{\mathrm{c}} = 0$, the dealer's total expected profit is obviously equal to zero for every β. Thus, it is optimal for the dealer to renege on the promised spread. When $T = \infty$, a non-cooperative phase lasts forever. This implies that $\Pi(S^*) = 0$ and for this reason $\Pi(S^* - 1)$ is finite for every β.

Proposition 4. *When the investor uses the cooperative trading policy* $q^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$, *the scoring policy* $p^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$ *is the optimal pricing policy for the relationship dealer if and only if* (a) $s^{\mathrm{c}} > 0$ *and* (b) $T < \infty$.

To sum up, in this section, we have shown that, for a given value of λ, the scoring strategy $p^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$ and the cooperative trading policy $q^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$ are self-enforcing iff (a) the no-informed trading condition is satisfied, (b) $s^{\mathrm{c}} > 0$ and (c) $T < \infty$.

Remarks.

(1) Observe that the cooperative trading policy is such that the investor never trades in both markets when she has a hedging need, as conjectured in Section 3. We establish that this is indeed optimal in the proof of Proposition 3.
(2) It is worth emphasizing that the no-informed trading condition imposes a cap on the investor's reputational capital. This cap guarantees that the investor's reputational capital is never so large that she can afford milking part of it. To see this point, observe that the no-informed trading condition cannot be satisfied when the maximal value of the score, S^*, is too large (because the L.H.S. of this condition is bounded). Intuitively, an increase in S^* defers the penalty associated with a breach of the no-informed trading agreement. For this reason, the cost of breaching the no-informed trading agreement at the very early stage of the cooperative phase becomes smaller and smaller as S^* increases. For a sufficiently large value of S^*, the investor is better off violating the no-informed trading agreement.
(3) One may wonder why the investor does not purposely trade at a loss against her regular dealer when she is informed in order to avoid non-cooperative phases. Observe that the regular dealer does not decrease the investor's score when he makes a profit. This implies that the continuation value of the relationship with the dealer is identical when the investor refrains from trading with the regular dealer ("strategy 1") or when she purposely trades at a loss with the regular dealer ("strategy 2"). Thus, strategy 1 always dominates strategy 2. To sum up, by design, the scoring strategy considered in this paper is "manipulation-free".

4.3. Cooperative equilibria

Observe that, *other things equal*, the no-informed trading condition is more easily satisfied when the risk of informed trading in the once-off dealer market, λ, becomes greater (the L.H.S. of the no-informed trading condition increases with λ). Actually, once-off dealers' spread increases with this risk. Thus, the investor's "outside option" deteriorates when λ increases. Accordingly, her incentive to honor the no-informed trading agreement is greater. The set of feasible no-informed trading agreements (i.e. the values of T, S^*, s^{c} sustaining an agreement) is therefore in part determined by λ. In turn, the probability of informed trading in the once-off dealer market is endogenous and is in part determined by the fraction of investors in position to get a price improvement. In order to formalize this point, let denote by ω^* the probability that the investor is in a cooperative phase and by $\lambda(\Psi, \omega^*)$ the resulting value for λ. We obtain the following result.

Corollary 1. *For a given no-informed trading agreement in the relationship market, the likelihood that an order received by once-off dealers comes from an informed trader is*:

$$\lambda(\Psi, \omega^*) = \frac{\alpha}{1-(1-\alpha)\Psi\omega^*}. \tag{15}$$

where ω^* is the probability that an investor with a relationship is in a cooperative phase. This probability is given by:

$$\omega^* = \left(\frac{\Lambda}{\Lambda + \mathrm{Prob}(\Delta S = +1)}\right) < 1, \tag{16}$$

where $\Lambda \stackrel{\mathrm{def}}{=} S^*/T$.

When no investor has a relationship, the probability of receiving an informed order for once-off dealers is $\lambda(0, 0) = \alpha$. This probability is larger when the relationship market is open as $\lambda(\Psi, \omega^*) > \alpha$ when $\omega^* > 0$ and $\Psi > 0$. Actually, the no-informed trading agreement allows the relationship dealer to divert uninformed orders from the once-off dealer market. This diversion increases the proportion of informed orders in the order flow routed to the once-off dealer market. This form of cream-skimming is common in financial markets (see Easley et al., 1996, Bessembinder & Kaufman, 1997or Barclay et al., 2003 for empirical findings). For instance, for Nasdaq stocks, Barclay et al. (2003) obtain several findings consistent with a scenario in which informed traders predominantly route their orders to ECNs whereas Nasdaq dealers retain the less informed orders. In particular, they find that realized spreads on Nasdaq are much larger than on ECNs.[22]

The risk of informed trading in the once-off dealer market depends on the characteristics of the no-informed trading agreement in the relationship market as λ increases with $\omega^* = S^*/T$. On the other hand, we already pointed out that the set of feasible agreements in the relationship market depends on the risk of informed trading in the once-off dealer market. Hence, the equilibrium of the relationship market and the once-off dealer market are *jointly* determined. A *cooperative equilibrium* is a situation in which the course of actions prescribed by a scoring policy and the associated cooperative trading policy are self-enforcing for $\lambda = \lambda^*(\Psi, \omega^*)$. This last condition guarantees that the equilibria in the once-off dealer market and in the relationship market are mutually consistent.

We immediately deduce from the analysis of the previous section (from Propositions 3 and 4) that the scoring policy $p^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$ and the cooperative trading policy $q^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$ form a cooperative equilibrium if and only if (a) the no-informed trading condition is satisfied for $\lambda = \lambda^*(\Psi, \omega^*)$ (b) $s^{\mathrm{c}} > 0$ and (c) $T < \infty$. For given values of the exogenous parameters, α and γ, the relationship market is *viable* if and only if there exists *at least one* cooperative equilibrium. The next result delineates the set of parameters for which the relationship market is viable.

Proposition 5. *For given values of α and γ, the relationship market is viable* (*i.e. at least one cooperative equilibrium exists*) *if and only if* $\gamma < \mathrm{Min}\,\{0.5, 2\alpha/(1+\alpha)\}$.

Region A in Fig. 3 is the set of values for parameters α and γ for which the relationship market is viable. In Region B, the relationship market is not viable : it is impossible to find a scoring policy that enforces a no-informed trading agreement. The intuition for this finding is as follows. As the investor's trading motivation cannot be observed, the relationship dealer bases his decision to grant a price improvement on his past trading profits with the investor. Past trading profits constitute an imperfect indicator of the investor's true behavior as the dealer can make a trading loss whether the investor behaves or not. For this reason, the score assigned to an investor can increase *even though the investor honors the no informed trading agreement*. This occurs

[22] This result fits well with our model where the realized spread in the relationship market is $s^{\mathrm{c}} > 0$ while the realized spread in the once-off dealer market is zero.

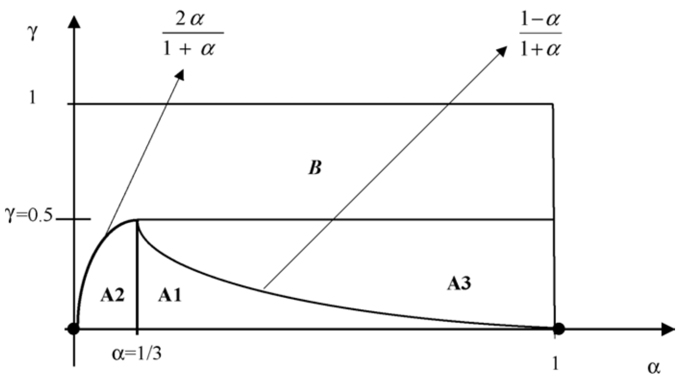

Fig. 3. Regions of existence for cooperative equilibria. For values of (γ, α) in Region A (A1, A2, A3), a cooperative equilibrium exists. In Region A3, the once-off dealer market is not viable, even when the relationship market is not open. In Region A1, lethal cooperative equilibria exist. That is, there exists a cooperative equilibrium for which the once-off dealer market is crowded out. For values of (γ, α) in Region B, it is impossible to enforce a no-informed trading agreement.

with probability $Prob(\Delta S = +1) = (1 - \alpha)/2 > 0$. This possibility of 'mistakenly' punishing the investor and the fact that S^* must be finite (see Remark 1 at the end of Section 4.2) imply that cooperative phases cannot last forever ($\omega^* < 1$). This effect bounds the value of establishing a relationship with the regular dealer and thereby the possibility of enforcing a no-informed trading agreement.

Observe that for a given value of α, the relationship market is not viable when the investor's risk aversion is too small ($\gamma \geq \mathrm{Min}\,\{0.5, 2\alpha/(1 + \alpha)\}$). Actually, risk sharing gains are smaller for traders with a small level of risk aversion. For this reason, these investors obtain a small per period utility gain from the no-informed trading agreement. Accordingly, these investors are not too much affected by a loss in their reputational capital and cannot be disciplined.

Example 1. We consider a numerical example to illustrate the results of this section. Suppose that $\alpha = 0.5$, $\gamma = 0.1$, $Q = 1$ and $\Psi = 0.7$. These parameter values are in Region A in Fig. 3. Thus, there exists a scoring policy sustaining a cooperative equilibrium (Proposition 5). Consider the following scoring policy : $S^* = T = 100$ (so that $\Lambda = S^*/T = 1$) and $s^c = 0.45$. In this case, the probability of observing a price improvement for an investor with a relationship is $\omega^* = 0.8$. It follows that the fraction of informed orders in the order flow routed to the once-off dealer market is $\lambda(0.7, 0.8) = 0.69$. For this specification, the L.H.S. of the no informed trading condition is $(\Delta U(0.1, 0.69))/(\bar{U}^c_{in}(0.1)) = 1.31$ and the R.H.S. is equal to 1.25. Thus, the no-informed trading condition is satisfied. Moreover, $s^c > 0$ and T is finite. We deduce that the scoring policy $p^{lt}(100, 100, 0.45)$ sustains a cooperative equilibrium. In this case, the bid-ask spread posted in the once-off dealer market is $s^{nc}(\lambda(\Psi, \omega^*)) = 0.69$.

5. Implications

5.1. The role of price improvements and other order routing inducements

The no informed trading condition imposes $\Delta U(s^c, \lambda(\Psi, \omega^*)) > 0$ (see Eq. (11)). This requires $s^c < \mathrm{Min}\{s^{nc}, \bar{s}\}$. Thus, the regular dealer *must* grant a price improvement to the investor during cooperative phases. Otherwise, there is no gain to being cooperative for the investor and the no informed trading condition cannot hold. The size of the price improvement has an ambiguous effect on incentives. On the one hand, a large price improvement (a small value of s^c) results in more efficient hedging for the investor. This effect increases the per period welfare gain in cooperative phases ($\Delta U(s^c, \lambda(\Psi, \omega^*))$) and contributes to discipline the investor. On the other hand, a large price improvement makes informed trading with the relationship dealer more profitable ($\bar{U}^c_{in}(s^c)$ decreases with s^c). This effect reduces the investor's incentive to behave. The first effect always dominates the second effect in our model (formally, the ratio $(\Delta U(s^c, \lambda(\Psi, \omega^*)))/(\bar{U}^c_{in}(s^c))$ decreases with s^c). This implies that the larger the price improvement, the greater is the incentive to cooperate for the investor, as stated by the next proposition.

Corollary 2. *For the investor to cooperate with the relationship dealer, she must receive a price improvement during cooperative phases. Furthermore, larger price improvements stiffens the investor's incentive to cooperate, i.e. f the no informed trading condition holds for a given price improvement* (*say* $s^c = s_0$) *then it holds true for larger price improvements* ($s^c < s_0$), *other things equal.*

Consequently, for given exogenous parameters, there exist multiple cooperative equilibria. For instance, we deduce from the previous corollary that, for the parameter values given in Example 1, all the scoring strategies such that $S^* = T = 100$ and $0 < s^c \leq 0.45$ form an equilibrium. The price improvement ultimately offered by the relationship dealer should depend on his relative bargaining power. We do not analyze further the determination of the price improvement as our results in this paper are independent of its exact size.

Interestingly, scoring strategies are such that a price improvement is always offered simultaneously with *a size improvement* as $q^c(s^c) > q^{nc}(\lambda)$ (recall that $q^c(s^c)$ is the trade size in the relationship market). Interestingly, dealers often offer both price and size improvements (Bacidore, Battalio, & Jennings, 2002 document this phenomenon for NYSE). Also, Bernhardt et al. (2005) find a positive correlation between the size of price improvements and trade size for a sample of FTSE-100 stocks (see their Table 1). In our model, this positive correlation arises naturally : the investor's hedging demand increases when the relationship dealer's bid-ask spread decreases (i.e. when the price improvement increases).

It is worth noting that price improvements are not the only form of monetary inducements that can enforce a no-informed trading agreement. Consider the following trading arrangement. During cooperatives phases, the relationship dealer offers to execute all orders for q^c shares at the quotes posted in the once-off dealer market with a "side" payment to the investor equal to $P = q^c * (s^{nc} - s^c)$. The two arrangements generate the same payoffs for the relationship dealer and the investor. Thus, direct monetary inducements could be used in lieu of price improvements to enforce no-informed trading agreements. Some market-makers do in fact offer side payments to brokerage firms in exchange for brokers routing investors' orders to them. This practice, known as *payment for order flow*, can be viewed as a substitute for price improvements.

5.2. Market design and welfare

In our model, the investor has access to two possible trading venues for the same security : (i) a market in which she can trade anonymously (the once-off dealer market) and (ii) a market in which she has an ongoing relationship with a specific dealer (the relationship market). Although this situation is frequently encountered in reality, it is unclear whether it is desirable or not. One problem, arising in our model, is that coexistence of multiple trading venues may lead to a situation in which informed orders are systematically routed to one trading venue, at the expense of its liquidity. This concern was voiced for instance in Easley et al. (1996).[23] They write pp. 831–832:

> Using a sample of stocks typically employed in purchased order flow, we have demonstrated that the information content of orders differs across trading locales [...] The fragmentation of orders by type can thus impose costs on existing markets well in excess of the effects arising purely from trade volume [...] It is not straightforward to determine the welfare effects of competition through order diversion [...] This issue requires further study and empirical testing.

Our model gives the possibility to make progress on this issue by comparing investors' welfare when the once-off dealer market operates alone ($\Psi = 0$) and when it coexists with the relationship market (which requires $\Psi > 0$). Throughout this section, we measure investors' welfare by their per-period expected utility. We focus on investors' welfare as once-off dealers get a zero expected profit anyway and the relationship dealer is obviously better off when the relationship market is open.

Investors who *cannot* establish a relationship trade only in the once-off dealer market. They obtain a per-period expected utility equal to[24]:

$$E\bar{U}^{\mathrm{nc}}(\lambda(\Psi,\omega^*)) \stackrel{\mathrm{def}}{=} \begin{cases} -(1-\alpha)\left(\dfrac{\bar{s}(\gamma)Q}{1+\bar{s}(\gamma)}\right) \quad \text{if} \quad \gamma \geq \dfrac{1-\lambda(\Psi,\omega^*)}{1+\lambda(\Psi,\omega^*)}, \\ \alpha\left(\dfrac{\gamma(1-s^{\mathrm{nc}}(\lambda(\Psi,\omega^*)))Q}{1+s^{\mathrm{nc}}(\lambda(\Psi,\omega^*))}\right) - (1-\alpha)\left(\dfrac{s^{\mathrm{nc}}(\lambda(\Psi,\omega^*))Q}{1+s^{\mathrm{nc}}(\lambda(\Psi,\omega^*))}\right) \\ \text{if} \quad \gamma < \dfrac{1-\lambda(\Psi,\omega^*)}{1+\lambda(\Psi,\omega^*)}. \end{cases}$$

The first line in the previous equation gives the investor's expected utility when the once-off dealer market is not viable (which happens when $\gamma \geq (1-\lambda(\Psi,\omega^*))/(1+\lambda(\Psi,\omega^*))$; see Section 3). The second line gives the the investor's expected utility when the once-off dealer market is viable. Now consider an investor who can establish a relationship. Her average *per-period* payoff is given by $(1-\beta)V(0)$.[25] Computations yield[26]:

$$\lim_{\beta\to 1}(1-\beta)V(0) = E\bar{U}^{\mathrm{nc}}(\lambda(\Psi,\omega^*)) + \omega^*\Delta U(s^{\mathrm{c}}, \lambda(\Psi,\omega^*)), \tag{17}$$

where the expression for $\Delta U(s^{\mathrm{c}}, \lambda(\Psi,\omega^*))$ is given in Proposition 3. Thus, the per period expected utility of an investor *with* a relationship is equal to (i) the per-period expected utility of an investor *without* a relationship plus (ii) the per period welfare gain brought up by cooperation times the probability that the investor and the regular dealer are in a cooperative phase. The next result is central for our findings in this section.

Corollary 3. *Consider a cooperative equilibrium in which the once-off dealer market and the relationship market coexist. The risk of informed trading* $(\lambda(\Psi,\omega^*))$ *and the bid-ask spread*

[23] See also Bessembinder and Kaufman (1997).

[24] This is also the per-period expected utility of an investor with a relationship during non-cooperative phases. This per-period expected utility is derived in the proof of Lemma 2 (Step 1).

[25] This is the total value of the relationship at the beginning of a cooperative phase ($V(0)$) divided by the expected number of periods until termination of the relationship ($1/(1-\beta)$).

[26] See the proof of Proposition 3 for a formal derivation.

charged by once-off dealers ($s^{nc}(\lambda(\Psi, \omega^*))$) *are larger than when the relationship market does not exist.*

The intuition for this result is straightforward. The relationship dealer diverts non-informed orders from the once-off dealer market. Accordingly, the fraction of informed orders in the order flow received by once-off dealers is larger when the relationship dealer is active than when it is not. This implies that the bid-ask spread in the once-off dealer market is wider when the relationship market is active. The diversion of uninformed trades by the relationship dealer results in a reduction of the liquidity of the once-off dealer market. In some cases, this reduction is so large that it *causes* a collapse of the once-off dealer market when it would otherwise be viable if the relationship market was closed. When this happens, we say that the cooperative equilibrium is "*lethal*". The condition under which lethal cooperative equilibria occur is given in the next corollary. Let $\Psi^* \stackrel{\text{def}}{=} ((1-\alpha) - \gamma(1+\alpha))/((1-2\gamma)(1-\alpha))$.

Corollary 4. *Suppose that the once-off dealer market is viable when it does not coexist with the relationship market* (*i.e.* $\gamma \leq (1-\alpha)/(1+\alpha)$). *There always exists at least one lethal cooperative equilibrium if* $\alpha \geq 1/3$ *and* $\Psi \geq \Psi^*$.

The region of values for (γ, α) for which the relationship market endangers the existence of the once-off dealer market is depicted in Fig. 3 (Region A1). The reason for the existence of lethal equilibria is simple. Introduction of the relationship dealer triggers an increase in the bid-ask spread charged by the once-off dealers. If the proportion of investors with a relationship is large enough ($\Psi \geq \Psi^*$), the increase in this bid-ask spread is so large that investors are better off not trading in the once-off dealers market, which then collapses.

Example 2. The exogenous parameters (but Ψ) are chosen as in Example 1 ($\alpha = 0.5$, $\gamma = 0.1$ and $Q = 1$). First, consider the case in which the once-off dealer market operates alone ($\Psi = 0$). In this case, $\lambda(0, 0) = 0.5$. The once-off dealer market is viable as $\gamma < (1 - \lambda(0, 0))/(1 + \lambda(0, 0))$ (see Proposition 1). The bid-ask spread charged in this case is $s^{nc}(\lambda(0.8, 0)) = 0.5$. Now suppose that the relationship market is open and that the proportion of investors with a relationship is $\Psi = 0.9$. Moreover, suppose that the regular dealer uses a scoring policy such that $S^* = 180$, $T = 100$, and $s^c = 0.1$. For this scoring policy, $\omega^* = 0.88$. It is easily checked that the scoring policy sustains a lethal cooperative equilibrium as (i) $\gamma > (1 - \lambda(0.9, 0.88))/(1 + \lambda(0.9, 0.88))$ (which implies that the once-off dealer market is not viable given the scoring policy used by the relationship dealer) and (ii) the no-informed trading condition is satisfied (computations show that the L.H.S. of this condition is equal to 2.2 whereas the R.H.S. is equal to 2.05).

Corollaries 3 and 4 imply that the relationship market is harmful for the investors *who cannot* establish a relationship when, in absence of a relationship market, the once-off dealer market is viable. More surprisingly, even investors *who can* establish a relationship may be worse off when the relationship market is open. The intuition is as follows. The creation of the relationship market has two opposite effects on the welfare of these investors. On the one hand, it allows investors to better hedge their risky endowment as they receive price improvements during cooperative phases. On the other hand, during non-cooperative phases, they trade at prices worse than those obtained when the relationship market does not exist. In some cases (lethal equilibria), the investors do not even trade during non-cooperative phases, whereas they could if the relationship market did not exist. This worsening of trading conditions during non-cooperative phases has a negative impact on *all* investors' welfare, including those with a relationship. The net impact of these two effects on these investors' welfare cannot be signed generally as it depends on parameter values. We illustrate this claim with the following example.

Example 3. Consider again Example 1. If the relationship market does not exist, the once-off dealer market is viable when it operates alone and the bid-ask spread charged in this case is $s^{\text{nc}}(\lambda(0, 0)) = \lambda(0, 0) = 0.5$ (see Example 2). The per period expected utility of an investor is then $E\bar{U}^{\text{nc}}(0.5) = -0.15$. When the relationship market is open, there is a cooperative equilibrium in which the regular dealer uses the scoring policy $p^{\text{lt}}(100, 100, 0.1)$. In this equilibrium, the once-off dealer market is viable and the bid-ask spread charged in this market is $s^{\text{nc}}(\lambda(\Psi, \omega^*)) = 0.69$. The per period expected utility of the investors without a relationship decreases and is equal to $E\bar{U}^{\text{nc}}(0.69) = -0.195$. The per period expected utility for investors with a relationship is:

$$\lim_{\beta\to 1}(1-\beta)V(0) = E\bar{U}^{\text{nc}}(0.69) + \omega^* \times \Delta U(0.1, 0.69)$$
$$= -0.195 + 0.8 \times 0.159 = -0.067.$$

In this case, investors who can establish a relationship benefit from the creation of the relationship market. Now, consider the cooperative equilibrium in which $S^* = T = 100$ and $s^{\text{c}} = 0.45$. In this case, the average ex ante per period expected utility for investors with a relationship is:

$$\lim_{\beta\to 1}(1-\beta)V(0) = E\bar{U}^{\text{nc}}(0.69) + \omega^* \times \Delta U(0.45, 0.69)$$
$$= -0.195 + 0.8 \times 0.04 = -0.156.$$

This is smaller than investors' expected utility when the relationship market does not exist as $E\bar{U}^{\text{nc}}(0.5) = -0.15$. This shows that even investors who can forge long-term relationships can be hurt by the creation of the relationship market.

Thus, when the once-off dealer market is viable *in absence of the relationship market* (i.e. when $\gamma < (1-\alpha)/(1+\alpha)$), the creation of the relationship market harms investors without a relationship and has an ambiguous effect on the remaining investors.[27] When the once-off dealer market is not viable *anyway* (i.e. when $\gamma \geq (1-\alpha)/(1+\alpha)$), the conclusion is different. In this case, investors who cannot establish a relationship are not affected by the creation of a relationship market, as they do not trade whether the relationship market exists or not. In contrast, investors who can establish a relationship are clearly better off when the relationship market is open as they can, at least partially, hedge their risky endowment.

The next corollary summarizes the welfare effects associated with the creation of the relationship market when the latter is viable.

Corollary 5. *Consider the cases in which the relationship market is viable. When* $\gamma \geq (1-\alpha)/(1+\alpha)$, *the creation of a relationship market improves the welfare of the investors who can establish a relationship and leaves other investors' welfare unchanged. When* $\gamma < (1-\alpha)/(1+$

[27] When all investors are hurt by the creation of the relationship market, why don't they commit not to use this market? The problem is that, individually, no investor has an incentive to honor this commitment. Suppose that all investors enter into such a commitment and are believed by the once-off dealers. In this case, the trading conditions in the once-off dealer market do not deteriorate as once-off dealers behave as if the relationship market was not active. Thus, the mechanism by which investors with a relationship can be hurt by the relationship market does not operate. But then, it becomes optimal for each investor to violate the commitment of not using the relationship market. Thus this commitment is not self-enforcing and cannot last.

α), *the creation of the relationship market diminishes the welfare of the investors who cannot establish a relationship and has an ambiguous impact on the welfare of the investors who can establish a relationship*.

The creation of a relationship market constitutes a Pareto improvement *only* when the once-off dealer market *is not viable* (Region A3 in Fig. 3). In this case, the relationship market facilitates trades that otherwise would not occur because the amount of adverse selection is too large. Otherwise (i.e. for values of the parameters in Regions A1 and A2 in Fig. 3), the creation of the relationship market always impairs the welfare of the investors without a relationship.

To sum up, the creation of the relationship market is a Pareto improvement when this is the only way to facilitate transactions (because the anonymous market structure is not viable). Otherwise, the creation of the relationship market harms at least one group of investors (those who cannot develop relationships) and in some cases *all* investors (even those who trade in the relationship market). These findings vindicate the concerns that have been raised about practices that allow dealers (or markets) to cream-skim uninformed orders. Of course, these practices may have other benefits not considered in this paper (such as speed of execution or increased competitive pressures on the primary market).

5.3. Empirical implications

The main and novel prediction of our model is that the decision to grant a price improvement by a dealer depends on the profitability of his past transactions with the client requesting the improvement. This hypothesis can be tested with data tracking trades of a particular dealer with each of his clients. Using data for a sample of 25 FTSE-100 stocks, Bernhardt et al. (2005) find a positive relationship between the dealer's profits with a specific brokerage firm and the price improvements granted to this firm, especially for small and large orders (see their Fig. 3).[28]

Bernhardt et al. (2005) also find a positive relationship between the trading volume of a specific broker and the price improvements received by this broker and propose a theoretical explanation for this relationship. The discounted stream of profits generated by a specific customer increases with her trading frequency. Thus, dealers are more willing to offer large price improvements to more frequent customers. Our model suggests a complementary explanation. Only investors who have frequent trading needs can be disciplined by the threat of losing price improvements (see the discussion following Eq. (10) in Section 4.2). This also creates a positive correlation between measures of trading volume and price improvements.

In each period, there is a probability $f_{\text{imp}} \stackrel{\text{def}}{=} (1-\alpha)\Psi\omega^*$ of observing a price improvement. For a given α, this probability increases with Ψ and ω^*. Observe also that $\lambda(\Psi, \omega^*)$ increases in ω^* and Ψ. Thus, α being fixed, the model predicts a positive relationship between the frequency at which price improvements take place and the likelihood of informed trading in the once-off dealer market.

Corollary 6. *Fixing the value of α, there is a positive correlation between the likelihood of informed trading in the once-off dealer market and the frequency with which price improvements occur in the relationship market. There is also a positive correlation between the size of the bid-*

[28] Battalio, Jennings, and Selway (2001) study the order flow routed to a major Nasdaq dealer. They find that the dealer's gross trading revenues and prices vary across routing brokers, after controlling for trade size. Our model suggests to explain these variations by dealer's profits with each brokerage firm.

ask spread in the once-off dealer market and the frequency with which price improvement occur in the relationship market.

A test of this prediction is not easy because α (the unconditional likelihood of informed trading) cannot be directly observed. However, this variable could be estimated using techniques developed in Easley et al. (1996) for instance. Also, notice that our model emphasizes the fact that ω^* and the bid-ask spread in the once-off dealer market are jointly determined (see Section 4.3). This implies that the frequency of price improvements and the bid-ask spread in the once-off dealer market are jointly determined. This endogeneity should also be taken into account in designing a test of the previous corollary.

6. Robustness

We have established three main results:

(1) A relationship dealer can enforce a no-informed trading agreement by granting price improvements contingent on his past trading profits.
(2) Hence, the relationship dealer can "cream-skim" uninformed trades. Cream-skimming exacerbates the adverse selection problem faced by once-off dealers and thereby result in large posted spreads in the once-off dealers market.
(3) For this reason, the creation of a relationship market is not Pareto improving when the once-off dealer market is viable.

Other reputation-based pricing policies. These results have been established by considering one specific type of pricing policy for the regular dealer (the scoring policy). This pricing policy is such that the investor has a fixed quota of losses that she can inflict to the regular dealer. If she exhausts this quota then the dealer stops granting price improvements for a while. Of course there are many other ways in which the prices offered by a dealer could depend on his trading profits with the investor. In fact there exist equilibria with more complex strategies for the dealer and the investor. A robust feature of these equilibria, however is that the regular dealer enforces the no-informed trading agreement by granting price improvements contingent on his past trading profits. The two other results follow from the possibility of enforcing a no-informed trading agreement but do not rely on the specific mechanism used to sustain this agreement.

Recall that for some parameter values (e.g. $\gamma > 0.5$), it is not possible to enforce the no-informed trading agreement with the type of pricing policies considered in this paper. In these cases, we cannot discard the possibility that there may exist other profit-based pricing policies that sustain cooperation. We suspect however that there will always be some parameter values for which cooperation cannot be enforced when the investor's trading motivation is not observed. For instance we have analyzed the case in which the dealer reinstates the investor's score at zero each time he books a profit with the investor. In this case, the set of parameters under which a cooperative equilibrium does exist is exactly as described in Proposition 5.

Multiple relationship dealers. We have shown that even investors with a relationship may be harmed by the introduction of a relationship market. One may wonder whether this is still the case with multiple relationship dealers. Observe that our result relies on the fact that, during non-cooperative phases or when she is informed, an investor with a relationship trades at once-off dealers' quotes. Hence, this investor is hurt by the decrease in the liquidity of this market. With multiple relationship dealers, an investor could try to avoid being denied price improvements by switching dealers at the end of non-cooperative phases. But a situation in which investors immediately receive price improvements from a new relationship dealer cannot sustain a cooperative equilibrium as investors bear no cost if they misbehave. Thus, in cooperative equilibria involving multiple relationship dealers, an investor must go through an "*observation phase*" before receiving price improvements. Moreover, for relationships to be stable, the duration of this "observation phase" should be such that no investor has an incentive to switch to a new relationship dealer.[29] This implies that, even with multiple relationship dealers, the investor would experience periods in which she is denied price improvements. Therefore, she would still be hurt by a widening of the bid-ask spread charged by once-off dealers. A full analysis of this case is left for future work.

7. Conclusions

The ability of some dealers to price discriminate between informed and uninformed orders is often used as an explanation for price improvements (see for instance the quotation from Barclay et al. (2003) in the introduction of this paper). Seppi (1990) and Benveniste et al. (1992) provide theoretical foundations for this explanation. They argue that repeated relationships allow dealers to avoid informed orders by threatening informed traders of future sanctions. Their argument, however, relies on the possibility for dealers to detect a cheating trader with a strictly positive probability.

We complement this approach by considering the realistic case in which investors' trading motivations are unobservable. We show that a dealer can still avoid informed orders, even though he is unable to tell apart informed and uninformed orders. To this end, the dealer must make his decision to grant price improvements to a specific client contingent on the profitability of his past transactions with this client. Intuitively, this profitability is indicative of the client's past trading motivations because trading losses are more likely if the client sometimes routes informed orders to the dealer. Hence, the dealer can discipline an investor by penalizing her when he incurs a trading loss. In our model, this penalty takes the form of a decrease in the investor's reputational capital. This decrease shortens the number of trades for which the investor is eligible for a price improvement. This is costly for the investor as price improvements allow the investor to better hedge.

We also analyze the welfare effects associated with the introduction of a "relationship market" (i.e. a trading mechanism that allows traders to forge long-term relationships). Enduring relationships are Pareto improving only when informational asymmetries preclude trading otherwise. Else, they worsen the liquidity of alternative trading venues. Through this channel, the welfare of investors who do not receive price improvements is impaired. Moreover, in some cases, even those who get price improvements suffer a loss in welfare.

The analysis generates some new predictions. Most important, it predicts a positive relationship between price improvements granted to a specific client by a dealer and the profitability of trades conducted with this client. Moreover, it predicts that stocks featuring a larger frequency of price improvements should have larger spreads. Last, it implies that only investors with a sufficiently large trading frequency will receive price improvements. This effect may encourage investors to trade more in order to secure price improvements or to use intermediaries (such as brokers).

[29] Thus, the duration of observation phases must exceed the duration of non-cooperative phases. Notice that it is in the common interest of the relationship dealers to wait before offering price improvements to new customers. Doing otherwise would undermine the possibility of enforcing no-informed trading agreements which are the source of profits for relationship dealers.

There are several possible extensions that could be considered in future work. In our model, the purpose of the implicit contract between the relationship dealer and the investor is to eliminate the risk of informed trading. The relationship dealer can also seek to acquire the investor's information. The form of the implicit contract that enforce truthful revelation of the investor's information in this case is an interesting issue, which we leave for future research.

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Appendix A

Proof of Lemma 1.

The proof of this lemma relies on two results given in Lemma 3 and 4 below.

Lemma 3. *The dealers accommodate equilibrium sell orders* (*resp. buy orders*) *at a discount* (*resp. at a markup*) *relative to the security's expected value. Formally, on the equilibrium path,* $p^{\text{nc}*}(q) \in [0, 1]$ *for every* $q > 0$ *and* $p^{\text{nc}*}(q) \in [-1, 0]$ *for every* $q < 0$.

Proof. We prove the lemma for a buy order only. For a sell order, the argument is similar. The price posted by once-off dealers is equal to the expected value of the asset (see Eq. (3)). Hence, it belongs to $[-1, +1]$. Thus an investor with bad news (resp. good news) never submits a buy (sell) order. It follows that a buy order signals that the investor's type is different from θ_{B}. Consequently, if $q > 0$, we have $p^{\text{nc}*}(q) = E_{\phi^{\text{nc}}}(\epsilon|q) \geq 0$ (where this expectation is well defined since q is on the equilibrium path). □

The next lemma shows that the investor's optimal order size when she is uninformed is as given in Fig. 2. Buy orders execute at a premium, $s \geq 0$ and sell orders execute a discount, $-s \leq 0$. Recall that $\overline{s}(\gamma) = (1-\gamma)/(1+\gamma)$. $\overline{s}(\gamma) = (1-\gamma)/(2-(1-\gamma)) < 1$.

Lemma 4.

(1) *At price* $0 \leq s \leq 1$, *the investor's optimal buy order when she has type* θ_{SH} *is*: (a) $Q/(1+s)$ *if* $s < \overline{s}(\gamma)$; (b) *zero if* $s > \overline{s}(\gamma)$; (c) *any quantity in* $\left[0, Q/(1+s)\right]$ if $s = \overline{s}(\gamma)$.
(2) *At price* $-1 \leq -s < 0$, *the investor strictly prefers to not trade rather than to submit a sell order when she has type* θ_{SH}.
(3) *At price* $-1 \leq -s \leq 0$, *the investor's optimal sell order when she has type* θ_{L} *is*: (a) $-Q/(1+s)$ *if* $s < \overline{s}(\gamma)$; (b) *zero if* $s > \overline{s}(\gamma)$; (c) *any quantity in* $\left[-Q/(1+s), 0\right]$ *if* $s = \overline{s}$.

(4) *At price* $0 < s \leq 1$, *the investor strictly prefers to not trade rather than to submit a buy order when she has type* θ_L.

Proof. The final wealth of an investor with type θ_{SH} when she trades q shares at price $p(q)$ is

$$W_\epsilon(q) = -Q\epsilon + q(\epsilon - p(q)). \tag{18}$$

Suppose first that $q \geq 0$ and $p(q) = +s$. In this case (i) $W_{-1}(q) \leq 0$ if and only if $q \geq Q/(1+s)$ and (ii) $W_{+1}(q) \leq 0$ if and only if $q \leq Q/(1-s)$. The investor's expected utility, $E(U(W_\epsilon(q)))$, is

$$E(U(W_\epsilon(q))) = \begin{cases} \frac{\gamma}{2}W_{-1}(q) + \frac{1}{2}W_{+1}(q) & \text{if} \quad 0 \leq q \leq \frac{Q}{1+s}, \\ \frac{1}{2}W_{-1}(q) + \frac{1}{2}W_{+1}(q) & \text{if} \quad \frac{Q}{1+s} \leq q \leq \frac{Q}{1-s}, \\ \frac{1}{2}W_{-1}(q) + \frac{\gamma}{2}W_{+1}(q) & \text{if} \quad q \geq \frac{Q}{1-s}. \end{cases} \tag{19}$$

It is immediate that $E(U(W_\epsilon(q)))$ is continuous and piecewise linear. When $0 \leq q \leq Q/(1+s)$, the slope of $E(U(W_\epsilon(q)))$ is positive if $s < \overline{s}(\gamma)$, zero if $s = \overline{s}(\gamma)$, negative if $s > \overline{s}(\gamma)$; When $q \geq Q/(1+s)$, the slope of $E(U(W_\epsilon(q)))$ is negative. Thus, for $q \geq 0$, $E(U(W_\epsilon(q)))$ reaches its maximum for $q = Q/(1+s)$ if $s < \overline{s}(\gamma)$ and for $q = 0$ if $s > \overline{s}(\gamma)$. For $s = \overline{s}(\gamma)$, any quantity in $\left[0, Q/(1+s)\right]$ maximizes $E(U(W_\epsilon(q)))$. This proves the first part of the lemma.

Now suppose that $q < 0$. Then $p(q) = -s$. *I*t immediately follows that (i) $W_{-1}(q) \geq 0$ and (ii) $W_{+1}(q) \leq 0$. Thus

$$E(U(W_\epsilon(q))) = \frac{\gamma}{2}W_{-1}(q) + \frac{1}{2}W_{+1}(q). \tag{20}$$

After some computations, we obtain

$$E(U(W_\epsilon(q))) = \frac{(\gamma-1)}{2}Q - \frac{(\gamma-1)}{2}q - \left(1 + \frac{(\gamma-1)}{2}\right)qs. \tag{21}$$

Using this expression and Eq. (19) written for $q = 0$, it is easily shown that

$$E(U(W_\epsilon(q))) < E(U(W_\epsilon(0))), \forall q < 0. \tag{22}$$

This proves the second part of the lemma. The third and the fourth parts of the lemma are proved using identical arguments. □

Armed with the two previous results we can now prove Lemma 1. Lemma 3 and 4 imply (i) $q^{nc*}(\theta_{SH}) \geq 0$, (ii) $q^{nc*}(\theta_L) \leq 0$, (iii) $q^{nc*}(\theta_G) \geq 0$ and (iv) $q^{nc*}(\theta_B) \leq 0$. Observe that either $q^{nc*}(\theta_G) \neq q^{nc*}(\theta_{SH})$ or $q^{nc*}(\theta_G) = q^{nc*}(\theta_{SH})$. Suppose $q^{nc*}(\theta_G) > 0$ and $q^{nc*}(\theta_G) \neq q^{nc*}(\theta_{SH})$ (to be contradicted). In this case, a buy order of size $q^{nc*}(\theta_G)$ reveals that the investor's type is θ_G. Hence, $p^{nc*}[q^{nc*}(\theta_G)] = E(\tilde{\epsilon}|\theta = \theta_G) = +1$. Thus, the investor obtains a zero expected profit when she has type θ_G. In this case she is indifferent between trading and not trading and we have assumed that she does not trade in such a situation. We conclude that $q^{nc*}(\theta_G) > 0$ and $q^{nc*}(\theta_G) \neq q^{nc*}(\theta_{SH})$ is impossible in equilibrium. Now assume that $q^{nc*}(\theta_G) = 0$ and $q^{nc*}(\theta_{SH}) > 0$ (to be contradicted). In equilibrium, $p^{nc*}[q^{nc*}(\theta_{SH})] = E(\tilde{\epsilon}|\theta = \theta_{SH}) = 0$. But then $q^{nc*}(\theta_G) = 0$ cannot be optimal as the investor can make a strictly positive expected profit by submitting an order for $q^{\text{nc}*}(\theta_{\text{SH}}) > 0$ shares when she has type θ_{G}. We conclude that $q^{\text{nc}*}(\theta_{\text{G}}) = q^{\text{nc}*}(\theta_{\text{SH}})$ in equilibrium. The same argument proves that $q^{\text{nc}*}(\theta_{\text{L}}) = q^{\text{nc}*}(\theta_{\text{B}})$ in equilibrium.

If $q^{\text{nc}*}(\theta_{\text{SH}}) > 0$, we deduce (using Bayes rule) that

$$p^{\text{nc}*}[q^{\text{nc}*}(\theta_{\text{SH}})] = E(\tilde{\epsilon}|\theta = \{\theta_{\text{SH}}, \theta_{\text{G}}\}) = s^{\text{nc}}(\lambda).$$

In the same way, if $q^{\text{nc}*}(\theta_{\text{L}}) < 0$, we obtain

$$p^{\text{nc}*}[q^{\text{nc}*}(\theta_{\text{L}})] = E(\tilde{\epsilon} \mid \theta \in \{\theta_{\text{L}}, \theta_{\text{B}}\}) = -s^{\text{nc}}(\lambda).$$

Proof of Proposition 1.

Step 1. Strategies $q^{\text{nc}*}(.)$ and $p^{\text{nc}*}(.)$ form an equilibrium.

Once-off dealers' prices for trade sizes on the equilibrium path follow from Lemma 1. For trade sizes out-of-the equilibrium path, dealers' beliefs ϕ^{nc} (and then dealers' pricing behavior) can be chosen arbitrarily. Here dealers' beliefs are chosen in such a way that they assign a probability 1 to the event $\{\epsilon = 1\}$ (resp. $\{\epsilon = -1\}$) when they receive a buy (resp. sell) order.

Consider now the problem faced by the investor. We know from Lemma 4 (see the proof of Lemma) that submitting a buy (resp. sell) order of size $\overline{q}(\lambda) = 1/(1 + s^{\text{nc}}(\lambda))$ is a best response when the investor is has type θ_{SH} (resp. θ_{L}). It is also immediate that submitting a buy (resp. sell) order of size $\overline{q}(\lambda) = 1/(1 + s^{\text{nc}}(\lambda))$ is a best response for the investor when she has type θ_{G} (resp. θ_{L}). Actually any other order result in either negative or zero profits for an informed investor given the pricing strategy followed by once-off dealers. Hence we have proved that the strategies given in Proposition 1 form an equilibrium when $s^{\text{nc}}(\lambda) \leq \overline{s}(\gamma)$. □

Proof of Proposition 2.

We first establish that the strategies given in the proposition form an equilibrium (step 1). Then we show that the equilibrium described in the proposition is the unique equilibrium when $s^{\text{nc}}(\lambda) > \overline{s}(\gamma)$ (step 2).

Step 1.

Using Lemma 4, we deduce that the investor's best response to the once-off dealers' strategy is to not trade when she is uninformed. When she is informed, the investor is indifferent between trading or not given once-off dealers' pricing strategy. Hence, not trading is optimal for the investor in all possible states. Given the investor's trading strategy, there is a zero probability that once-off dealers receive an order. Thus, conditional on such an event, we can choose dealers' beliefs ϕ^{nc} (and then dealers' pricing behavior) arbitrarily. Here dealers' beliefs are chosen in such a way that they assign a probability equal to one to the event $\{\epsilon = 1\}$ (resp. $\{\epsilon = -1\}$) when they receive a buy (resp. sell) order.

Step 2.

Suppose (to be contradicted) that there is another equilibrium in which the investor trades, i.e. $q^{\text{nc}*}(\theta_{\text{L}}) \neq 0$ or/and $q^{\text{nc}*}(\theta_S) \neq 0$. In this case, Lemma 1 implies that $q^{\text{nc}*}(\theta_{\text{L}}) < 0$ or/and $q^{\text{nc}*}(\theta_S) > 0$. Furthermore, the lemma states that, in equilibrium, these orders are executed at price $s^{\text{nc}}(\lambda)$ (buy order) or $-s^{\text{nc}}(\lambda)$ (sell order). But, when $s^{\text{nc}}(\lambda) > \overline{s}(\gamma)$, the uninformed investor is better off not trading (see Lemma 4). This implies that $q^{\text{nc}*}(\theta_{\text{L}}) < 0$ or $q^{\text{nc}*}(\theta_{\text{SH}}) > 0$ cannot be a best response when $s^{\text{nc}}(\lambda) > \overline{s}(\gamma)$. A contradiction. We deduce that in this case there cannot exist an equilibrium in which trades take place in the once-off dealers market. □

Proof of Lemma 2.

Recall that we consider an investor with a relationship.

Step 1. Let $E\bar{U}^{\mathrm{c}}(\lambda, s^{\mathrm{c}})$ be the investor's *per period* expected utility during a cooperative phase. Similarly let $E\bar{U}^{\mathrm{nc}}(\lambda)$ be the investor's per period expected utility during a non-cooperative phase. By definition :

$$E\bar{U}^{\mathrm{c}}(\lambda, s^{\mathrm{c}}) \stackrel{\mathrm{def}}{=} (1-\alpha)\bar{U}^{\mathrm{c}}_{he}(\lambda, s^{\mathrm{c}}) + \alpha\bar{U}^{\mathrm{nc}}_{in}(\lambda, s^{\mathrm{c}}), \tag{23}$$

where $\bar{U}^{\mathrm{c}}_{he}(\lambda, s^{\mathrm{c}})$ denotes the investor's expected utility when she needs to hedge during a cooperative phase and $\bar{U}^{\mathrm{nc}}_{in}(\lambda, s^{\mathrm{c}})$ denotes the investor's expected utility when she is informed during a cooperative phase. When she needs to hedge (type θ_{SH} or θ_{L}), the investor trades $Q/(1+s^{\mathrm{c}})$ shares with her relationship dealer. As the latter charges a bid-ask spread equal to s^{c}, computations show that she obtains an expected utility equal to:

$$\bar{U}^{\mathrm{c}}_{he}(\lambda, s^{\mathrm{c}}) = \frac{1}{2}\left(-Q + \frac{Q(1-s^{\mathrm{c}})}{1+s^{\mathrm{c}}}\right) = \frac{-Qs^{\mathrm{c}}}{1+s^{\mathrm{c}}}. \tag{24}$$

When the investor is informed, she trades with once-off dealers when she follows the cooperative trading policy. Thus :

$$E\bar{U}^{\mathrm{nc}}_{in}(\lambda, s^{\mathrm{c}}) = \begin{array}{l} \dfrac{\gamma(1-s^{\mathrm{nc}}(\lambda))Q}{1+s^{\mathrm{nc}}(\lambda)} \quad \text{if} \quad \gamma < \dfrac{1-\lambda}{1+\lambda} \\[2ex] 0 \quad \text{if} \quad \gamma \geq \dfrac{1-\lambda}{1+\lambda} \end{array} \tag{25}$$

We deduce that:

$$E\bar{U}^{\mathrm{c}}(\lambda, s^{\mathrm{c}}) = \begin{cases} -(1-\alpha)\left(\dfrac{s^{\mathrm{c}}Q}{1+s^{\mathrm{c}}}\right) \quad \text{if} \quad \gamma \geq \dfrac{1-\lambda}{1+\lambda}, \\[2ex] \alpha\left(\dfrac{\gamma(1-s^{\mathrm{nc}}(\lambda))Q}{1+s^{\mathrm{nc}}(\lambda)}\right) - (1-\alpha)\left(\dfrac{s^{\mathrm{c}}Q}{1+s^{\mathrm{c}}}\right) \quad \text{if} \quad \gamma < \dfrac{1-\lambda}{1+\lambda}. \end{cases} \tag{26}$$

During non-cooperative phases, the investor trades only with once-off dealers. When the investor needs to hedge, she obtains an expected utility equal to

$$\bar{U}^{\mathrm{nc}}_{he}(\lambda) = \begin{array}{l} -\left(\dfrac{s^{\mathrm{nc}}(\lambda)Q}{1+s^{\mathrm{nc}}(\lambda)}\right) \quad \text{if} \quad \gamma < \dfrac{1-\lambda}{1+\lambda} \\[2ex] -\dfrac{\bar{s}(\gamma)Q}{1+\bar{s}(\gamma)} \quad \text{if} \quad \gamma \geq \dfrac{1-\lambda}{1+\lambda} \end{array} \tag{27}$$

When she is informed, the investor obtains an expected utility given by Eq. (25). Thus,

$$E\bar{U}^{\mathrm{nc}}(\lambda) = \begin{array}{l} -(1-\alpha)\left(\dfrac{\bar{s}(\gamma)Q}{1+\bar{s}(\gamma)}\right) \quad \text{if} \quad \gamma \geq \dfrac{1-\lambda}{1+\lambda}, \\[2ex] \alpha\left(\dfrac{\gamma(1-s^{\mathrm{nc}}(\lambda))Q}{1+s^{\mathrm{nc}}(\lambda)}\right) - (1-\alpha)\left(\dfrac{s^{\mathrm{nc}}(\lambda)Q}{1+s^{\mathrm{nc}}(\lambda)}\right) \quad \text{if} \quad \gamma < \dfrac{1-\lambda}{1+\lambda}. \end{array} \tag{28}$$

Let $\Delta U(s^{\mathrm{c}}, \lambda) = E\bar{U}^{\mathrm{c}}(\lambda, s^{\mathrm{c}}) - E\bar{U}^{\mathrm{nc}}(\lambda)$. We obtain :

$$\Delta U(s^{\mathrm{c}}, \lambda) = \begin{cases} \dfrac{(\bar{s}(\gamma) - s^{\mathrm{c}})(1-\alpha)Q}{(1+\bar{s}(\gamma))(1+s^{\mathrm{c}})} & \text{if} \quad \gamma \geq \dfrac{1-\lambda}{1+\lambda}, \\ \dfrac{(s^{\mathrm{nc}}(\lambda) - s^{\mathrm{c}})(1-\alpha)Q}{(1+s^{\mathrm{nc}}(\lambda))(1+s^{\mathrm{c}})} & \text{if} \quad \gamma < \dfrac{1-\lambda}{1+\lambda}. \end{cases} \tag{29}$$

As $s^{\mathrm{c}} < \mathrm{Min}\{s^{\mathrm{nc}}, \bar{s}\}$, it is straightforward that $\Delta U(s^{\mathrm{c}}, \lambda) > 0$, i.e. $E\bar{U}^{\mathrm{c}}(\lambda, s^{\mathrm{c}}) > E\bar{U}^{\mathrm{nc}}(\lambda)$.

Step 2. In cooperative phases, when the investor receives good or bad news, she only trades with once-off dealers. Thus

$$V(in, S) = \bar{U}_{in}^{\mathrm{nc}} + \beta V(S) \quad \textit{for} \quad S \leq S^* - 1, \tag{30}$$

When she needs to hedge, she trades with her relationship dealer and obtains a per period expected utility equal to $\bar{U}_{he}^{\mathrm{c}}$. Following this transaction, the investor's score deteriorates if the relationship dealer loses money. The probability of this event is 1/2. Otherwise her score is unchanged. Therefore,

$$V(he, S) = \bar{U}_{he}^{\mathrm{c}} + \beta\left(\frac{1}{2}V(S+1) + \frac{1}{2}V(S)\right) \quad \text{for} \quad S \leq S^* - 1. \tag{31}$$

Substituting Eq. (8) in the expressions of $V(in, S)$ and $V(he, S)$ given by Eqs. (30) and (31) respectively and rearranging, we obtain

$$V(S+1) = kV(S) - \frac{2E\bar{U}^{\mathrm{c}}}{\beta(1-\alpha)} \quad \text{for} \quad S \leq S^* - 1, \tag{32}$$

with $k \stackrel{\text{def}}{=} (1 + (2(1-\beta))/(\beta(1-\alpha)))$. Now, we deduce from Eq. (32) that

$$V(S+1) - V(S) = k(V(S) - V(S-1)) \quad \text{for} \quad 1 \leq S \leq S^* - 1. \tag{33}$$

Furthermore, Eq. (32) for $S = 0$ implies

$$V(1) - V(0) = (k-1)\left(V(0) - \frac{E\bar{U}^{\mathrm{c}}}{1-\beta}\right). \tag{34}$$

Recall that $V(0)$ is the (discounted at rate β) sum of the investor's per period payoff , which is either $E\bar{U}^{\mathrm{c}}$ or $E\bar{U}^{\mathrm{nc}}$. As $E\bar{U}^{\mathrm{c}} > E\bar{U}^{\mathrm{nc}}$ (see step 1), we obtain that

$$V(0) \leq \sum_{t=0}^{+\infty} \beta^t E\bar{U}^{\mathrm{c}} = \frac{E\bar{U}^{\mathrm{c}}}{1-\beta}. \tag{35}$$

Thus, Eq. (34) implies then that $V(1) - V(0) \leq 0$. In turn, this yields (iterating Eq. (33)):

$$V(S+1) - V(S) \leq 0 \quad \text{for} \quad 1 \leq S \leq S^* - 1. \tag{36}$$

Hence, $V(.)$ decreases with S. Furthermore, since $k > 1$ and $V(0) - V(1) \geq 0$, Eq. (33) implies that $\Delta V(S) = (V(S-1) - V(S))$ increases for $1 \leq S \leq S^*$. □

Proof of Proposition 3.

This proposition is a direct implication of Lemma 5 and 6 below.

Lemma 5. *The cooperative trading policy* $q^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$ *is optimal when the dealer follows the scoring policy* $p^{\mathrm{lt}}(T, S^*, s^{\mathrm{c}})$ *if and only if*

$$\beta(\Delta V(0)) \geq \bar{U}^{\mathrm{c}}_{in}(s^{\mathrm{c}}). \tag{37}$$

Proof. According to the Optimality Principle of Dynamic Programming, the investor has no incentive to deviate from the cooperative trading strategy q^{lt} if and only if there is no circumstance in which a one shot deviation is profitable.[30] Hence, we just need to show that the condition given in Lemma A is necessary and sufficient to deter a one shot deviation from the trading strategy q^{lt}. It derives immediately from the analysis of Section 3 that $q^{\mathrm{nc}*}(.)$ gives the investor's optimal order with once-off dealers. It follows that no one shot deviation is profitable during non-cooperative phases. During cooperative phases, it is straightforward that $q^{\mathrm{c}} = Q/(1 + s^{\mathrm{c}})$ is the investor's optimal order size when she trades with her relationship dealer. The one shot deviations that remain to analyze are:

(1) The investor is informed with good (bad) news. She sends a buy (sell) order to her relationship dealer (instead of contacting only the once-off dealers).
(2) The investor is uninformed. She trades with the relationship dealer *and* the once-off dealer (instead of trading only with the relationship dealer).
(3) The investor is uninformed and she trades only with the once-off dealer (instead of trading with the relationship dealer).

We consider each of these actions in turn.

Deviation 1. We have already explained in the text that the first deviation is not optimal if and only if Condition (37) holds true.

Deviation 2. When the investor is uninformed, the quantity traded with the regular dealer is the investor's optimal trade size given the price charged by this dealer (see Lemma 4). This means that trading a larger size at the regular dealer's spread (s^{c}) is suboptimal. This in turn implies that routing another order to the once-off dealer market, is suboptimal. Hence, deviation 2 is suboptimal.

Deviation 3. When the investor is uninformed and she has score $S \leq S^* - 1$, she is better off trading with her relationship dealer if and only if

$$\bar{U}^{\mathrm{nc}}_{he} + \beta V(S) \leq V(he, S), \tag{38}$$

($\bar{U}^{\mathrm{nc}}_{he}$ is given in Eq. (27)). Using Eq. (31), we rewrite Condition (38) as

$$\bar{U}^{\mathrm{nc}}_{he} \leq \bar{U}^{\mathrm{c}}_{he} + \frac{\beta}{2}(V(S+1) - V(S)), \tag{39}$$

($\bar{U}^{\mathrm{c}}_{he}$ is given in Eq. (24)). Using Eq. (32), after straightforward manipulations, we can rewrite Eq. (39) as

$$\frac{E\bar{U}^{\mathrm{nc}}}{1-\beta} \leq V(S), \tag{40}$$

($E\bar{U}^{\mathrm{nc}}$ is given in Eq. (28)). Notice that $V(S)$ is the discounted sum of the investor's per period payoff (which is either $E\bar{U}^{\mathrm{c}}$ or $E\bar{U}^{\mathrm{nc}}$). As $E\bar{U}^{\mathrm{c}} \geq E\bar{U}^{\mathrm{nc}}$ (see Step 1 in the proof of Lemma 2),

[30] As the relationship dealer's strategy is Markovian (the state variable S_t follows a Markov chain, see the second part of the proof of this proposition), the investor's equilibrium strategy is Markovian and contingent on S_t as well.

we deduce that

$$V(S) \geq \sum_{t=0}^{+\infty} \beta^t E\bar{U}^{\mathrm{nc}} = \frac{E\bar{U}^{\mathrm{nc}}}{1-\beta}, \tag{41}$$

which means that Eq. (40) holds true. Hence, deviation 3 is suboptimal. □

Lemma 6. *When* β *goes to* 1, *the condition* $\beta(\Delta V(0)) \geq \bar{U}_{in}^{\mathrm{c}}(s^{\mathrm{c}})$ *can be written* $(\Delta U(s^{\mathrm{c}}, \lambda))/(\bar{U}_{in}^{\mathrm{c}}(s^{\mathrm{c}})) \geq ((1-\alpha)/2 + S^*/2T)$.

Proof. Denote by $E\tilde{U}_t$ and by S_t the investor's expected utility and score in period t when she follows the cooperative trading policy. $E\tilde{U}_t$ depends on whether the relationship is in a cooperative or a non-cooperative phase. If $S_t \in [0, S^* - 1]$ then $E\tilde{U}_t = E\bar{U}^{\mathrm{c}}$. If $S_t \in [S^*, S^* + T - 1]$ then $E\tilde{U}_t = E\bar{U}^{\mathrm{nc}}$ ($E\bar{U}^{\mathrm{c}}$ and $E\bar{U}^{\mathrm{nc}}$ are given in Eqs. (26) and (28)). Notice that S_t is a Markov chain with state space $\{0, 1, \ldots, S^* + T - 1\}$. Denote by $\mathrm{Pr}_{SS'}$ the transition probability from state S to state S'. We have (i) for $S \leq S^* - 1$, $\mathrm{Pr}_{S(S+1)} = 0.5(1-\alpha)$ and $\mathrm{Pr}_{SS} = 1 - 0.5(1-\alpha)$; (ii) for $S^* \leq S < S^* + T - 1$, $\mathrm{Pr}_{S(S+1)} = 1$ and $\mathrm{Pr}_{(S^*+T-1)0} = 1$; (iii) the other transition probabilities are zero. Let $o(S)$ be the long-run frequency of state S. By definition, the probabilities $o(S)$ form the invariant measure of the Markov chain, namely the solution of the linear system:

$$\forall S', o(S') = \sum_{S=0}^{S=S^*+T-1} \Pr_{SS'} o(S). \tag{42}$$

Given that most of the $\mathrm{Pr}_{SS'}$ are zero, solving for $o(S)$ is easy. In particular, we obtain

$$o(S) = \frac{1}{S^* + T(1-\alpha)/2}, \quad \forall S \in [0, S^* - 1].$$

We deduce that the probability, ω^*, of a cooperative phase is

$$\omega^* \stackrel{\mathrm{def}}{=} \sum_{S=0}^{S=S^*-1} o(S) = \frac{S^*/T}{(S^*/T) + (1-\alpha)/2}. \tag{43}$$

Observe that ω^* and $1-\omega^*$ give the long-run frequencies of $E\bar{U}^{\mathrm{c}}$ and $E\bar{U}^{\mathrm{nc}}$ respectively. An ergodic theorem for Markov chains (Theorem I.15.2 in Chung, 1967) implies that

$$\lim_{\tau \to +\infty} \frac{\sum_{t=0}^{t=\tau} E\tilde{U}_t}{T+1} = \omega^* E\bar{U}^{\mathrm{c}} + (1-\omega^*) E\bar{U}^{\mathrm{nc}} \quad \text{with probability} \quad 1 \tag{44}$$

Denoted by $E\bar{U}_t$ the expected value of $E\tilde{U}_t$ conditional on the score being equal to zero at a given date $t_0 < t$ (say $t_0 = 0$), i.e. $E\bar{U}_t = E[E\tilde{U}_t | S_0 = 0]$. For $\tau \geq 0$ and $\beta \in [0, 1]$, define $f_\tau(\beta) \stackrel{\mathrm{def}}{=} \left(\sum_{t=0}^{t=\tau} \beta^t E\bar{U}_t\right) / \left(\sum_{t=0}^{t=\tau} \beta^t\right)$. Notice that

$$f_\infty(\beta) = (1-\beta)V(0). \tag{45}$$

Every f_τ is a continuous function of β. Furthermore, when τ tends to $+\infty$, we have pointwise convergence of $f_\tau(\beta)$ to $f_\infty(\beta) \stackrel{\mathrm{def}}{=} (\sum_{t=0}^{t=\infty} \beta^t E\bar{U}_t)/(\sum_{t=0}^{t=\infty} \beta^t)$. Dini's Theorem implies then that convergence of f_τ to f_∞ is uniform on the compact set $[0, 1]$. Hence, f_∞ is continuous at $\beta = 1$. Using this remark, Eqs. (44) and (45), we conclude that:

$$\lim_{\beta \to 1} (1-\beta)V(0) = f_\infty(1) = \omega^* E\bar{U}^{\mathrm{c}} + (1-\omega^*) E\bar{U}^{\mathrm{nc}}. \tag{46}$$

Now recall that $\Delta V(0)$ is given by Eq. (34). Substituting $(\Delta V(0))$ by its expression in $\beta(\Delta V(0)) \geq \bar{U}^{c}_{in}(s^{c})$ gives

$$\frac{2}{(1-\alpha)}\left(E\bar{U}^{c} - (1-\beta)V(0)\right) \geq \bar{U}^{c}_{in}(s^{c}). \tag{47}$$

Substituting $(1-\beta)V(0)$ by its expression given in Eq. (46) (recall that we consider the limit case in which β goes to 1), we rewrite the previous equation as

$$E\bar{U}^{c} - E\bar{U}^{nc} \geq \bar{U}^{c}_{in}(s^{c})\left(\frac{(1-\alpha)}{2} + \frac{S^{*}}{2T}\right). \tag{48}$$

The L.H.S of this inequality is, by definition, $\Delta U(s^{c}, \lambda)$ (the expression for this expected utility differential has been derived in Lemma 2, Step 1). We deduce that the no-informed trading condition is equivalent to:

$$\frac{\Delta U(s^{c}, \lambda)}{\bar{U}^{c}_{in}(s^{c})} \geq \left(\frac{(1-\alpha)}{2} + \frac{S^{*}}{2T}\right). \quad \square$$

Proof of Proposition 4.

The investor's trading policy is Markovian (it is determined by S_t that follows a Markov chain). Consequently, the dealer's equilibrium strategy is Markovian and determined by S_t as well. Therefore, according to the Optimality Principle of Dynamic Programming, it is sufficient to check that, for each value of the score, no one-shot deviation is profitable. For $S < S^{*}$, we have shown in the text that the dealer should not charge a spread different from s^{c} when she receives an order of size q^{c}. For trade sizes out-of-the equilibrium path, the regular dealers' beliefs ϕ^{c} can be chosen arbitrarily. Here the beliefs are specified in such a way that the dealer assigns a probability 1 to the event $\{\epsilon = 1\}$ (resp. $\{\epsilon = -1\}$) when he receives a buy (resp. sell) order. The same reasoning yields the dealer's pricing strategy when $S \geq S^{*}$ (in this case any order is out-of-the equilibrium path as the investor does not contact the dealer during non-cooperative phases). $\square$

Proof of Corollary 1.

Recall that $\lambda(\Psi, \omega^{*})$ is the probability that an order received by once-off dealers is informed. Thus, by definition:

$$\lambda(\Psi, \omega^{*}) = \text{Prob}[\theta \in \{\theta_{B}, \theta_{G}, \theta_{N}\} | q(\tilde{\theta}, \tilde{S}) = (0, q^{nc*}(\tilde{\theta}))]$$

The investor chooses to trade with the relationship dealer if (1) she has a relationship, (2) she is in a cooperative phase and (3) she is uninformed. This event occurs with probability $\Psi\omega^{*}(1-\alpha)$. Hence, the investor routes her order to the once-off dealer's market with probability $(1 - \Psi\omega^{*}(1-\alpha))$. We deduce that:

$$\begin{aligned}&\text{Prob}[\theta \in \{\theta_{B}, \theta_{G}, \theta_{N}\} | q(\tilde{\theta}, \tilde{S}) = (0, q^{nc*}(\tilde{\theta}))] \\ &\quad = \frac{\text{Prob}[\theta \in \{\theta_{B}, \theta_{G}, \theta_{N}\} \cap q(\tilde{\theta}, \tilde{S}) = (0, q^{nc*}(\tilde{\theta}))]}{\text{Prob}[q(\tilde{\theta}, \tilde{S}) = (0, q^{nc*}(\tilde{\theta}))]}, = \frac{\alpha}{(1 - \Psi\omega^{*}(1-\alpha))}.\end{aligned}$$

The expression for ω^{*} has been derived in the proof of Lemma 6(see proof of Proposition 3, Eq. (43)). $\square$

Proof of Proposition 5.

The proof of this result is tedious. We omit it for brevity. It is available upon request.

Proof of Corollary 2.

The first claim follows from the discussion that precedes the proposition. In order to prove the second claim, we define

$$G(s^{c}) \stackrel{\text{def}}{=} \Delta U(s^{c}, \lambda(\Psi, \omega^{*})) - \frac{(1-s^{c})\gamma Q}{1+s^{c}}(\text{Prob}(\Delta S = +1) + \Lambda). \tag{49}$$

The no-informed trading condition (Condition (11)) is equivalent to $G(s^{c}) \geq 0$. Using Eq. (12) for $\Delta U(s^{c}, \lambda(\Psi, \omega^{*}))$, we obtain that $G(s^{c})$ decreases with s^{c}. Thus, *other things equal*, if the no-informed trading condition is satisfied for $s^{c} = s_0$ then it is also satisfied for $s^{c} < s_0$. This establishes the proposition. □

Proof of Corollary 3.

Observe that λ increases with Ψ and ω^{*}. This implies $\lambda(\Psi, \omega^{*}) > \lambda(0, 0)$. Consequently, $s^{nc}(\lambda(\Psi, \omega^{*})) > s^{nc}(\lambda(0, 0))$. □

Proof of Corollary 4.

The once-off dealer market breaks-down if:

$$\gamma \geq \frac{1-\lambda(\Psi, \omega^{*})}{1+\lambda(\Psi, \omega^{*})}. \tag{50}$$

In this case the no-informed trading condition imposes:

$$\frac{(\overline{s}(\gamma) - s^{c})(1-\alpha)Q}{(1+\overline{s}(\gamma))(1+s^{c})} - \frac{\gamma(1-s^{c})Q}{1+s^{c}}\left(\frac{(1-\alpha)}{2} + \Lambda\right) \geq 0. \tag{51}$$

Consider the following scoring strategy : $s^{c} \simeq 0$, $\Lambda = ((1-2\gamma)/\gamma)((1-\alpha)/2)$. It is easily shown that Conditions (50) and (51) are satisfied for this parameterization of the scoring strategy if $\Psi^{*} \leq \Psi \leq 1$. Moreover, this interval is non-empty (i.e. $\Psi^{*} \leq 1$) if $\alpha \geq 1/3$. Thus, the scoring strategy $s^{c} \simeq 0$, $\Lambda = ((1-2\gamma)/\gamma)((1-\alpha)/2)$ sustains a cooperative equilibrium in which the once-off dealers market is inactive. Note that if $\gamma < (1-\alpha)/(1+\alpha)$ then $\Psi^{*} > 0$, which means that in absence of the relationship market, the once-off dealer market would be viable. □

Proof of Corollary 5.

The corollary derives from the argument in the text. Example 3 shows that the creation of the relationship market has an ambiguous impact on the welfare of the investors with a relationship. □

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